Energy efficient heat sink design: Natural vs. forced convection cooling

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Energy Efficient Heat Sink Design: Natural vs. Forced Convection Cooling

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Abstract—In highly efficient converter systems the power consumption of the cooling system (fans) significantly influences the total system efficiency as well as the power density. This paper investigates the potential of free convection cooled heat sinks and compares their performance to forced convection cooled heat sinks on the basis of their volume and power consumption. Underlying theoretical concepts for both types of cooling systems are summarized and validated with prototype heat sinks. Finally, it is shown that by free convection cooling not only the efficiency but also the power density can be increased in specific cases. In this case by paralleling semiconductor devices the losses decrease approximately by a factor of three while simultaneously the heat sink volume can be reduced by approximately 50%.

I. INTRODUCTION

In power electronic systems, semiconductor devices are one of the major loss sources and the required cooling system contributes significantly to the total volume. Nevertheless, due to the reduced switching losses of modern power semiconductors (e.g. Si/SiC MOSFETs & GaN HEMTs), system efficiencies up to 99.5% can be achieved. In such highly efficient converters, the cooling system not only has an impact on the overall volume but also influences the total system efficiency in case of forced air cooling.

As a consequence, state of the art modelling concepts for forced convection cooling, used to optimize the heat sink with respect to its volume [1], [2] or the weight [3], are complemented with the power required for the cooling system (e.g. fans in this paper).

Due to the high system efficiency also natural convection cooling becomes an attractive option since in such highly efficient converter systems the required thermal resistance could be relatively large and accordingly natural convection heat sinks are sufficient. Furthermore, convection cooling could be an attractive solution from a volume point of view since no fan is required. In order to investigate the performance boundary between forced and natural convection cooling an optimization procedure for natural convection heat sinks based on the modeling concepts presented in [4], [5] is derived.

In addition, the trade off between the thermal resistance of the heat sink (heat sink volume & fan losses) and the temperature dependent conduction losses of the semiconductor devices is investigated with the heat sink optimization procedure presented in this paper. Section II recapitulates the theoretical background for forced convection cooling, which is extended by the fan losses (section II-A) and free (natural) convection cooling (section II-B). The resulting limitations of forced convection cooling as well as the potential of free convection cooling are discussed in section III based on two case studies of half-bridge buck-boost converter designs. The theoretical models are experimentally validated and the measurement results are shown in section IV.

II. HEAT SINK MODELLING

Generally the losses generated in the semiconductor devices \( P_{\text{semi}} \) (total conduction & switching losses) have to be dissipated to the ambient via a heat sink which significantly affects the required system volume. The maximal allowed thermal resistance of the heat sink \( R_{\text{th,hs,max}} \) to keep the junction temperature \( T_j \) of the semiconductors below the maximum...
allowed device temperature $T_{j,max}$ is calculated by

$$R_{th,hs,max} = \frac{\Delta T_{j-h, max}}{P_{dev, max} (R_{th, j-c} + R_{th, c-hs}) - T_{amb}}$$  

(1)

based on the maximum losses occurring in a single device $P_{dev, max}$ at the considered ambient temperature $T_{amb}$ see Fig. 1).

In the optimization procedure for air cooled heat sinks, $R_{th,hs,max}$ is the limiting parameter to achieve a thermally feasible design.

There are basically two types of air cooling: forced air cooling and free (natural) convection cooling. For both types of cooling, the thermal resistance of the heat sink $R_{th,hs}$ is optimized based on analytical and empirical equations which take the geometrical parameters (heat sink width $w_{hs}$, length $l_{hs}$, fin height $c_{hs}$, channel width $s_{hs}$, and fin width $t_{hs}$, Fig. 2) into account. In the following, the underlying analytical models presented in [3], [5] are briefly summarized and extended.

For both types of cooling, the lower limit of the baseplate area $A_b$ is defined by the number of semiconductor devices which have to be mounted on the heat sink. Furthermore, to achieve a small stray inductance in the half-bridge leg, the area $A_{L,par}$ has to be minimized (see Fig. 1b). This results in a minimal basic cell area $A_{HB,TO247} \approx 20 \text{mm} \times 60 \text{mm}$ of which multiples have to be placed on the heat sink in dependence of the number of parallelized half-bridge cells $n_{semi}$. To account for heat spreading effects in the baseplate, the baseplate area is limited to a maximum of twice the semiconductor area.

$$n_{semi} \cdot A_{HB,TO247} < A_b < 2 \cdot n_{semi} \cdot A_{HB,TO247}$$  

(2)

The physical values used in the following considerations of air cooled systems are listed in Table I.

**TABLE I. PHYSICAL VALUES.**

<table>
<thead>
<tr>
<th>Physical Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal conductivity of air ($\lambda_{air}$)</td>
<td>0.03 W/(m K)</td>
</tr>
<tr>
<td>Kinematic viscosity of air ($\nu_{air}$)</td>
<td>2.1 \times 10^{-5} m^2/s</td>
</tr>
<tr>
<td>Density of air ($\rho_{air}$)</td>
<td>1 kg/m^3</td>
</tr>
<tr>
<td>Expansion coefficient ($\beta_{air}$)</td>
<td>3.2 \times 10^{-3} K^{-1}</td>
</tr>
<tr>
<td>Thermal capacitance of air ($c_{air}$)</td>
<td>1.01 \times 10^{3} J/(kg K)</td>
</tr>
<tr>
<td>Prandtl number for air ($Pr$)</td>
<td>0.71 @ 60°C</td>
</tr>
<tr>
<td>Thermal conductivity (Al) ($\lambda_{hs}$)</td>
<td>210 W/(m K)</td>
</tr>
<tr>
<td>Emissivity Al, anodize black ($\epsilon_{hs}$)</td>
<td>≈ 0.95, polished ≈ 0.05</td>
</tr>
<tr>
<td>Stefan-Boltzmann constant ($\sigma$)</td>
<td>5.6703 \times 10^{-8} W/(m K^4)</td>
</tr>
<tr>
<td>Gravitational acceleration ($g$)</td>
<td>9.81 m/s^2</td>
</tr>
</tbody>
</table>

**A. Forced Convection Cooling**

Thermal heat sink optimization procedures for forced convection heat sinks (see Fig. 2) are presented in [1]–[3]. There, the three-dimensional problem is reduced to a two-dimensional problem based on a thermal network analysis and under the assumption of equally distributed power losses on the baseplate. Based on the work presented in [6], [7], the authors of [3] improved the work presented in [1] and derived a new procedure, where the thermal resistance $R_{th,hs}$ is determined based on the Nusselt number $Nu_T$, which characterizes the thermal transition from solid to fluid matter. The underlying analytical expressions can be found in [3].

Referring to the analysis presented in [3] a strong dependency of the thermal resistance $R_{th,hs}$ on the air volume flow $V$ through the channels and thus on the operating point of the fan can be observed. The operating point of the fan is determined by intersecting the characteristics of the heat sink and the fan (see Fig. 3a)). To determine the $\Delta p$- $V$ characteristic of the heat sink, several effects causing a pressure drop $\Delta p$ have to be considered [3], [8] including the pressure drop due to the fluid acceleration, the friction of the channel and sudden contraction/expansion at the heat sink inlet and outlet. The pressure drop in the heat sink channels with hydraulic diameter $d_h = (2s_{hs}c_{hs})/(s_{hs} + c_{hs})$ is determined according to [8] by

$$\Delta p_{hs} = \left( f_{fr} \frac{l_{hs}}{d_h} + K_{se} + K_{sc} \right) \cdot \frac{\rho_{air} V^2}{2}.$$  

(3)

For the cross section adapter the pressure drop calculates to

$$\Delta p_{ad} = \left( f_{fr,ad} \frac{l_{ad}}{d_{hs,ad}} + 1 \right) \cdot \frac{\rho_{air} V^2}{2}.$$  

(4)

and due to the acceleration of the fluid

$$\Delta p_{acc} = \left( \frac{1}{(n_{fan} - 1)s_{hs}c_{hs}} - \frac{1}{(h_{fan}w_{hs})^2} \right) \cdot \frac{\rho_{air} V^2}{2}.$$  

(5)

The explanations and expressions for the loss and friction factors ($K_{sc}$, $K_{sc}$, $K_{vent}$, $f_{fr}$, and $f_{fr,ad}$) can be found in [3].
By summing up the different pressure drops the overall \( \Delta p_{\text{sum}} \) of a specific heat sink results:

\[
\Delta p_{\text{sum}} = \Delta p_{hs} + \Delta p_{ad} + \Delta p_{acc} \approx K_{\text{tot}} \cdot \dot{V}^2. \tag{6}
\]

There, the pressure drop of the cooling system increases approximately quadratically with increasing volumetric flow. The general loss factor \( K_{\text{tot}} \) combines the different loss factors discussed above. Fig. 3 a) illustrates the two heat sink characteristics for the prototype systems shown in section IV.

There are several dependencies of the the fan \( \Delta p - \dot{V} \) characteristic which have to be taken into account, like rotor blade angle, fan geometry and speed. Since the according geometrical data is not always available, this paper considers only fans of the size 40mm \( \times \) 40mm \( \times \) 28mm from different manufacturers (Delta Electronics, NMB, Sanyo Denki) which are compact and provide a high diversity concerning speed, static pressure head and volume flow. Three typical fan characteristics are shown in Fig. 3a). The characteristic of geometrical similar fans scale with the fan laws and/or affinity laws [9], [10] what was proven based on datasheet values, where \( m \) is the fan speed.

According to these laws a quadratic dependency of the static pressure head on the volume flow can be observed. Since the \( \Delta p - \dot{V} \) characteristic of a specific heat sink also shows this quadratic behaviour (see (6)), the operating point of the cooling system is always on the same relative position on the fan characteristic in dependence of the speed (see intersecting points of the characteristics \( o_1 - o_3 \) in Fig. 3a).

The mechanical power \( P_m = \Delta p \cdot \dot{V} \) varies to the power of three referring to the fan speed. This mechanical power is linked by an efficiency coefficient \( \eta_{fan} \) to the electrical power consumption of the fan.

\[
P_{fan,el} = \eta_{fan} \cdot P_{m,\text{max}}. \tag{10}
\]

To determine \( \eta_{fan} \), datasheets of several fans have been analyzed (Fig. 2b)). However, the fan efficiency varies relatively strong and drops with decreasing fan speed. In the optimization procedure the highest achievable fan efficiency has been considered.

Referring to the laws (7)-(9) and assuming an approximately constant power consumption of the fan at a certain fan speed [11] (e.g. \( f_{c1} \) in Fig. 3a)), allows to recalculate the \( R_{th,hs} \) in dependence of the power consumption of the fan for a specific heat sink geometry (e.g. Fig. 3: \( o_1 - o_1'' \), \( o_3'' \)). Fig. 3b) illustrates the \( \dot{V} \) dependent power consumption of a 40mm \( \times \) 40mm \( \times \) 28mm fan and the resulting thermal resistance for the two prototype systems presented in section IV. Furthermore, it can be seen that high fan power levels (10W-20W) do not significantly affect the thermal resistance anymore while at low fan powers the sensitivity of the \( R_{th,hs} \) on \( P_{fan} \) is relatively high.

With the above presented analytical approach a \( R_{th,\dot{V}} - P_{fan} \) Pareto surface area can be calculated. Fig. 6a) exemplarily illustrates such a Pareto surface area for a half-bridge where two switches are connected in parallel. The limitation with respect to volume is defined by the area required to mount the devices, and the applied fan, which defines the width and the maximum height of the heat sink.

**B. Free Convection Cooling**

Without fan, the heat is transferred by free convection to the ambient. In [4], [5], [12], [13] analytical models for vertical parallel finned heat sinks, as illustrated in Fig. 4, are derived. There, an uniform fin temperature is assumed due to the relatively low thermal resistance inside the heat sink. The heat-transfer coefficient for natural convection \( \alpha_{nc} \) is related to the Nusselt number \( N_{Uf} \) based on the characteristic length \( \Gamma \) which characterizes a certain geometry by

\[
N_{Uf} = \frac{\alpha_{nc} \Gamma}{\lambda_{air}}. \tag{11}
\]
Based on the characteristic numbers $Ra_T$ and $Gr_T$, $Nu_T$ is given by

$$Nu_T = \frac{Ra_T}{\Psi} \left( 1 - \exp \left( -\Psi \left( \frac{0.5}{Ra_T} \right)^{3/4} \right) \right)$$  \hspace{1cm} (15)

$$\Psi = \frac{24(1 - 0.483e^{-17/\epsilon})}{(1+\frac{\epsilon}{2})(1+(1-e^{-0.83\epsilon})(0.14e^{1/2}e^{-464\Gamma_{hs}} - 0.61))^{3/4}}$$  \hspace{1cm} (16)

where $\epsilon$ is the channel aspect ratio. The surface area of the finned heat sink is

$$A_{sur, fin} = (n_{fin} - 1)(s_{hs} + 2c_{hs})l_{hs}$$  \hspace{1cm} (17)

The resulting thermal resistance of the finned part of the heat sink for the natural convection is

$$R_{th, fin} = \frac{1}{\alpha_{nc} A_{sur, fin}} = \frac{\Gamma}{Nu_T \lambda_{air} A_{sur, fin}}.$$  \hspace{1cm} (18)

Other parts of the heat sink or devices are also exposed to the ambient and thus have to be considered in the calculation. Most of these surfaces are arranged in a horizontal or vertical way and can be approximated by flat plates. The corresponding equations to estimate the thermal convection of vertically arranged plates with a constant surface temperature [5] are

$$Nu_{pl} = \left( \frac{(Nu_l)^m + (Nu_{pl})^m}{2} \right)^{1/m}$$  \hspace{1cm} (19)

with

$$Nu_l = \frac{2}{\ln(1 + 2/Nu_T)} = \frac{2}{\ln(1 + 2/(C_l \cdot Ra^{1/4}))}$$  \hspace{1cm} (20)

$$Nu_{pl} = \frac{C_l Ra^{1/3}}{1 + 1.4 \cdot 10^9 \cdot Pr/Ra}$$  \hspace{1cm} (21)

$$Ra = \frac{g \beta_{air} Pr \Delta T l_{pl}^3}{\nu_{air}^2}$$  \hspace{1cm} (22)

$$m = 6 \quad C_l = 0.103 \quad C_l = 0.515.$$  

The thermal resistance of a vertical plate with length $l_{pl}$ (vertical dimension) is given by

$$R_{th, pl} = \frac{1}{A_{pl} Nu_{pl} \lambda_{air} l_{pl}} = \frac{1}{w_{pl} l_{pl}^2 Nu_{pl} \lambda_{air}}.$$  \hspace{1cm} (23)

Similar equations for the horizontal plate arrangement can be found in [5]. Since in most cases the arrangement of the electronics is unknown it is assumed that only the finned part of the heat sink is exposed to the ambient the plate model is neglected in the optimization. However, the measurements in section IV show a reduction of up to 10% of $R_{th, tot}$ due to additional convective elements.

Commercially available heat sinks are often black anodized to improve the heat transfer by radiation due to the increased emissivity $\epsilon$. The surface of the finned part of the heat sink is approximated by

$$A_{rad} = 2 \cdot (l_{hs}(c_{hs} + d_{hs}) + w_{hs}d_{hs}) + l_{hs}w_{hs}.$$  \hspace{1cm} (24)

In arbitrary setups additional radiating surfaces have to be considered (e.g. semiconductor devices). Corresponding to the
Boltzmann law, the thermal resistance due to the radiation \( R_{\text{th,rad}} \) is

\[
R_{\text{th,rad}} = \frac{\Delta T}{Q} = \frac{\Delta T}{c_{\text{hs}} \sigma A_{\text{rad}} (T_{\text{hs}}^4 - T_{\text{amb}}^4)}
\]

(25)

where temperatures are in Kelvin. The parallel connection of \( R_{\text{th,fin}} \), \( R_{\text{th,pl}} \) and \( R_{\text{th,rad}} \) results in the total thermal resistance of the heat sink

\[
R_{\text{th,tot}} = \left( \frac{1}{R_{\text{th,fin}}} + \frac{1}{R_{\text{th,pl}}} + \frac{1}{R_{\text{th,rad}}} \right)^{-1}.
\]

(26)

Within the optimization a general dependency of the thermal resistance on the heat sink volume can be observed which is in good accordance with the \( R_{\text{th}} \) achievable by commercially available free convection heat sinks (see Fig. 4b).

\[
R_{\text{th,nc}} = 0.4633 \frac{K}{W^{3/5}} \cdot 10^{-3}, V^{-0.9306} + 0.122 \frac{K}{W}
\]

(27)

However, the thermal resistance is strongly dependent on the ambient temperature as well as on the temperature difference (see Fig. 4c) since with increasing temperature on one hand the air acceleration and thus the thermal transfer coefficient are improving and on the other hand the radiation is increasing to the power of four.

### III. Optimization Results

In the following, two cases based on a half-bridge buck-boost converter (see Fig. 1, Table II) are investigated to compare the different thermal solutions. The underlying optimization procedure based on the previously discussed analytical models is described in Fig. 5. Case I: General investigation of forced convection cooling heat sinks and Case II: Possible volume and power density improvement by free convection cooling heat sinks. The half-bridge buck-boost converter is built with SiC MOSFETs (CMF20120D). The geometric design space for the optimization is chosen based on commercially available heat sinks (see Table III [14]).

#### A. Case I

Fig. 6a) depicts the achievable \( R_{\text{th,hs}} \) as function of the fan power \( P_{\text{fan}} \), and the total heat sink volume \( V_{\text{hs}} \) (including the volume of the heat sink, the fan, and the inlet adapter) based on the optimization procedure described in Fig. 5. As exemplarily shown in Fig. 6a) on the Pareto curve for a chosen \( R_{\text{th,hs}} = 0.5 \text{K/W} \) the required volume of the heat sink \( V_{\text{hs}} \) becomes smaller by increasing the applied \( P_{\text{fan}} \). Such a Pareto curve results for every \( R_{\text{th,hs}} \) with \( P_{\text{fan}} \) and \( V_{\text{hs}} \) as parameters. However, as already can be seen in Fig. 3b) especially at high fan powers, only small improvements of the \( R_{\text{th}} \) can be observed. A higher number of parallel connected semiconductor devices would increase the required \( R_{\text{th,hs}} \), because the losses would decrease. Referring to the example at hand (\( P_{\text{fan}} \) vs \( V_{\text{hs}} \), gray plane Fig. 6a)) the \( R_{\text{th,hs}} \)-plane can be raised and due to increased number of semiconductor devices the mechanical limit is shifted to higher volumes, whereas the Pareto curves for the different \( P_{\text{fan}} \) have to be recalculated.

If the overall system efficiency is considered, in addition to \( P_{\text{fan}} \) also the losses of the semiconductors have to be taken into account. The conduction losses in the semiconductor devices are temperature dependent (e.g. for the considered MOSFETs, \( R_{\text{ref}} = 80 \text{m}\Omega @ 60^\circ \text{C} \))

\[
R_{\text{ds,on}}(T_j) = R_{\text{ref}} \left( 0.988 + 4.65 \times 10^{-5} T_j + 8.54 \times 10^{-8} T_j^3 \right)
\]

(28)

Since colder devices are more efficient, a smaller \( R_{\text{th,hs}} \) enables lower semiconductor losses. For a fixed heat sink

### Table II: Operating Parameters

| Input voltage \( V_{\text{in}} \) | 400V |
| Output voltage \( V_{\text{out}} \) | 320V |
| Duty cycle \( D \) | 0.8 |
| Average inductor current \( I_l \) | 20A |
| Current ripple \( \Delta I_l \) | 10A |
| Switching frequency \( f_S \) | 50kHz |
B. Case II:

By increasing the required $R_{th,hs}$ (e.g., lowering the losses by paralleling semiconductor devices) not only the minimum required fan power decreases and thus the overall system efficiency increases, but also free convection heat sinks are sufficient. Fig. 7 depicts the maximum allowed thermal resistance $R_{th,hs,max}$ for the heat sink as function of the number of paralleled semiconductor devices in a half-bridge converter in buck operation (Fig. 1 & Table II). As can be seen, by only paralleling two devices, a natural convection heat sink would already be sufficient since the conduction losses of a single device are reduced by a factor of $\approx 4$. Referring to Fig. 6a) this is approximately the intersection point of the mechanical limit and the free convection and actually states the boundary where natural convection becomes beneficial in terms of volume compared to forced convection.

Fig. 7. a) Represents the losses of a single device $P_{dev}$ and the total semiconductor losses $P_{tot}$ of the half-bridge for the operating point in Table II as function of $n_{semi}$ and the resulting $R_{th,hs,max}$. b) Depicts the volume of the heat sink to achieve the required $R_{th,hs}$ for natural ($V_{nc}$) and forced ($V_{fc}$) convection heat sinks. The circle marks the intersection point between natural convection and the mechanical limit (see Fig. 6). For $n_{semi} = 3...5$ an efficiency and a power density gain for natural convection can be observed. By paralleling more than five devices $V_{nc}$ is increasing since it is determined by the area required to mount the devices and not by $R_{th,hs,max}$ ($A_{semi}$ driven). Symbols (1)-(3) indicate the realized prototype heat sinks given in section IV.

TABLE III

<table>
<thead>
<tr>
<th>Heat sink width</th>
<th>$w_{hs}$</th>
<th>[50mm ... 200mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat sink length</td>
<td>$l_{hs}$</td>
<td>[30mm ... 200mm]</td>
</tr>
<tr>
<td>Baseplate thickness</td>
<td>$d_{hs}$</td>
<td>3mm</td>
</tr>
<tr>
<td>Fin height</td>
<td>$h_{fs}$</td>
<td>[10mm ... 50mm]</td>
</tr>
<tr>
<td>Channel width</td>
<td>$s_{ch}$</td>
<td>[2mm ... 10mm]</td>
</tr>
<tr>
<td>Fin width</td>
<td>$h_{fs}$</td>
<td>[1mm ... 5mm]</td>
</tr>
<tr>
<td>Adapter length</td>
<td>$l_{ad}$</td>
<td>10mm</td>
</tr>
<tr>
<td>Fan dimensions</td>
<td>$w_f \times l_f \times d_f$</td>
<td>40mm×40mm×28mm</td>
</tr>
</tbody>
</table>

Fig. 6. a) Shows the boundaries for the minimal $R_{th,hs}$ that can be achieved in dependence of the applied fan power $P_{fan}$ and the mechanical constraints (Table III). Furthermore for $R_{th,hs} = 0.5 K/W$ the dependency between volume and $P_{fan}$ is illustrated (black). The circle marks the intersection between mechanical and free convection limit. If a $R_{th}$ higher than this value is required free convection enables a reduced volume (indicated red). b) Illustrates the overall losses of a half-bridge taking the thermal behaviour of the on-state resistance into account in dependency of the volume and the fan power. For every volume an optimum between applied fan power and loss reduction due to colder semiconductor devices can be found. Exemplarily for a heat sink volume of 0.1dm$^3$ (gray plane) the highest system efficiency is achieved with a fan with 3W power consumption.

Volume $V_{hs}$ can be decreased by using a fan with a higher $P_{fan}$. For the optimization procedure in Fig. 5 steps $n_1-n_3$ & $f_1-f_2$ have to be modified by an internal iteration loop. In this loop the losses of the semiconductors have to be recalculated for the actual calculated temperatures ($T_j$ & $T_{hs}$) and for each temperature until the temperature deviation between the iterations is negligible. For natural convection cooling also $R_{th,hs}$ is temperature dependent. As a consequence $n_1-n_2$ have to be iterated to determine the according losses and thermal resistances.

Taking the overall losses ($P_{tot} = P_{fan} + P_{semi}$) into account, an optimal heat sink design minimizing $P_{tot}$ can be found as indicated in Fig. 6b). Obviously, fans with a comparably high power do not always result in an optimal heat sink design since their power consumption is higher than what is gained by the lower junction temperature of the semiconductors.
If three devices are paralleled not only the system efficiency can be improved but also the volume of the natural convection heat sink becomes smaller than the minimal volume achieved by forced convection. Parallelising more than five devices would result in a higher volume because of the increased mounting area required for the semiconductors $A_{semi}$ (indicated in Fig. 7 as $A_{semi}$ driven). Additionally, increasing losses $P_{tot}$ can be observed by parallelising more than seven devices due to increasing switching losses (sum of all switching losses). The offset between the volume curves ($V_{fc}$ & $V_{nc}$ in Fig. 7) indicates the volume lost due to the mounted fan and due to the mechanical limits in this consideration. If the overall efficiency is taken into account and only the upper boundary of the junction temperature $T_{j,max}$ is fixed, as discussed in case I, the efficiency can be improved by applying fans due to the reduced $R_{ds,on}$ of colder devices. In Fig. 8 this effect is taken into account to determine the optimal cooling system for different number of devices, whereas $T_{amb} < T_j < T_{j,max} = 100°C$. By parallelising three devices the efficiency gain because of the reduced device temperature is approximately equal or less than the fan power consumption and the total losses for forced and natural convection are approximately superimposed. Since for natural convection heat sinks no space for the fan and the adapter is required the volume can be further reduced. This results in an efficiency and power density gain for natural convection compared to forced convection as indicated by $\Delta A$ → $\Delta B$ in Fig. 8. However, this gain is always liked to an increasing amount of semiconductor devices and thus with higher costs and complexity.

**IV. Validation of the Models**

To validate the analytical heat sink models and the results for Case II (section III Fig. 7) four prototype heat sinks were built. The dimensions can be given in Fig. 10 and Fig. 9. To generate the heat flux in the heat sinks two respectively six series connected diodes (IDW40E65D1) have been mounted and supplied with a constant forward current. Since these diodes have a negative temperature coefficient in the considered current range, the temperature of the devices/ on the heat sink is self-stabilizing. The temperatures of the ambient and of the heat sink base plate were measured with standard type-K thermocouples. The results for the forced convection cooling can be found in Table IV. The thermal resistance fits quite well to the analytical results. However, the power consumption of the fans is increased compared to the optimal determined power consumption due to the significant variance of the fan efficiency (see Fig. 3). Due to the comparably high thermal resistance of the free convection heat sink, for the measurement additional convecting and radiating surfaces have to be considered (see Fig. 10c). In the measurement setup the semiconductors build also a finned structure and the devices are radiating ($R_{th,rad}$). Thermal tests include blank and anodized heat sink in which the fins are vertically (see Fig. 10b) and horizontal upward arranged where the heat sink and the ambient temperature as well as the power losses of the diodes were measured. The measurements were executed for about 30 minutes per measurement point to reach the thermal equilibrium. The measurement results are summarized in Table V.

For all test setups a reduction of the $R_{th,tot}$ with increas-
Fig. 10. a) Represents the geometrical dimensions of the prototype system for natural convection (a). There, one heat sink is black anodized and the other blank. b) Depicts the measurement setup. In c) the additional considered convecting and radiating surfaces are indicated.

TABLE V
MEASUREMENT RESULTS FOR NATURAL CONVECTION COOLING FOR CASE STUDY

<table>
<thead>
<tr>
<th>$T_{amb}$ °C</th>
<th>$T_{hs}$ °C</th>
<th>$P_{semis}$ W</th>
<th>$R_{th,m}$ K/W</th>
<th>$R_{th,tot}$ K/W</th>
<th>$R_{th,hs}$ K/W</th>
<th>$R_{th,ad}$ K/W</th>
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<td>blank, vertically arranged</td>
<td>blank, vertically arranged</td>
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<td>4.05</td>
<td>5.28</td>
<td>17.38</td>
</tr>
<tr>
<td>black anodized, horizontally arranged</td>
<td>black anodized, horizontally arranged</td>
<td></td>
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</tr>
<tr>
<td>24.8</td>
<td>62.5</td>
<td>8.94</td>
<td>4.21</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>25.0</td>
<td>71.1</td>
<td>11.30</td>
<td>4.07</td>
<td>-</td>
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</tbody>
</table>

The calculations with a deviation of less than 5% was observed for the considered prototypes. With the models, the theoretical limits of forced and free convection cooling are discussed based on the example of a half-bridge converter. There, also the temperature dependent on-state resistance of the devices is taken into account. In specific cases the efficiency as well as the power density of the overall system can be improved by employing natural convection heat sinks instead of forced convection. In the considered case, the losses can be reduced by factor of 3 and the corresponding heat sink volume can be reduced by 50% by paralleling semiconductor devices. However, to enable a thermally feasible design employing natural convection, a high amount of semiconductor devices is required and its performance strongly depends on the occurring temperatures (ambient, heat sink temperature) and on the installation position. Thus, natural convection heat sinks are an attractive solution for fixed installations (e.g. solar farms, EV charging stations), where the heat sinks can be arranged such that the air flow is not influenced by other heat sources and a high efficiency is required. For installations in a cabinet (e.g. server stations), where no thermal decoupling can be assumed, a shared fan at the inlet or outlet of the cabinet could be sufficient, since already a small air flow significantly reduces the thermal resistance.

V. CONCLUSION

In highly efficient converter systems the power consumption of the cooling system could have a significant impact on the overall system efficiency. Especially for highly efficient systems it is important to include the power consumption of the fans in the analysis since high power fans do not necessarily result in the highest efficient heat sink design. Therefore, in this paper the theoretical models for forced convection cooling are extended by the power of the fan. Furthermore, a modeling and optimization concept for free convection cooling is presented and validated with prototype systems. A good accordance between the measurements and the calculations is achieved for natural convection.

REFERENCES