Long Horizon, Quadratic Programming Based Model Predictive Control (MPC) for Grid Connected Modular Multilevel Converters (MMC)

S. Fuchs, M. Jeong, J. Biela
Power Electronic Systems Laboratory, ETH Zürich
Physikstrasse 3, 8092 Zürich, Switzerland
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Abstract—Excellent control performance is a prerequisite to operate compact modular multilevel converter (MMC) designs that employ very small module capacitance values. This paper presents a novel model predictive control (MPC) scheme featuring a linear and accurate prediction model to overcome the drawbacks of comparable control schemes. The MPC is based on a modulator that determines the switching states of the individual modules to guarantee scalability of the algorithm also for MMCs with a high number of modules. The control performance of the proposed MPC scheme is evaluated with simulations and compared to a classic cascaded PI-controller scheme.

Index Terms—Modular Multilevel Converter (MMC, M2C), Model Predictive Control (MPC), Circulating current control, Energy balancing

I. INTRODUCTION

After more than a decade since its invention [1], the Modular Multilevel Converter (MMC) represents one of the standard topologies for converters that operate at high voltages. It is applied in many application fields in high voltage as well as in medium voltage systems [2], [3]. A major drawback of the MMC is its need for relatively large module capacitance values, such that these significantly contribute to the MMC’s volume, weight and cost. To ensure fast transient handling of MMCs without exceeding the maximum allowed module voltage, the minimum required module capacitance values [4] are usually scaled with a safety factor [5]. Alternatively, the module capacitor ripple voltage is limited to a certain percentage of its mean value. This is done to create a control margin for the inner arm voltage (sum of all module capacitor voltages of an MMC arm) as shown in Fig. 1. Instead of increasing the MMC’s volume and cost with larger capacitance values, high performance energy balancing control of the MMC could be used to keep the capacitor voltages within their bounds even with small safety factors and/or high ripple voltages, while keeping transient control performance at the highest physically feasible level.

Because of the multi-input multi-output (MIMO) characteristic of the MMC, the well known standard cascaded PI-controller structure (cf. [6]) results in relatively low transient performance. Also time delays due to sensing, communication and/or computation cannot be properly compensated with cascaded PI-controllers. For overcoming these limitations, MIMO control schemes have been proposed [7]. However, all these control schemes cannot consider system constraints like the maximum output voltage of the individual MMC arms, the maximum module voltage or arm/grid/DC currents. This can result in saturation and/or too high capacitor voltages after output power step commands.

The control of MIMO systems with constraints is a typical application area of Model Predictive Control (MPC) algorithms. MPC algorithms perform an online prediction of the system’s future behaviour and optimize the control input to find a tradeoff between tracking the (possibly contradicting) control references and the system constraints.

For MMCs, MPC could be beneficial, because one could decrease cost, volume and weight by minimizing the inductance and capacitance values without decreasing (transient) performance, as the MPC algorithm takes the system’s constraints into account. The MPC can predict that e.g. the module voltages might exceed their maximum value constraints in...
the future. As a reaction to this, the MPC would generate a counteracting circulating current, such that the output currents are affected as little as possible and an optimal trade off between meeting the constraints and tracking the reference can be implemented. Utilizing this MPC feature requires relatively long prediction horizons.

Many predictive control schemes have been proposed for the MMC in the literature. Most of them consider the switching state of each module as the system input [8]–[10]. This results in a large integer optimization problem, such that in most cases only one prediction step is considered to not cause a too heavy computational burden. More advanced control schemes as [11]–[14] manage to reduce the computational burden allowing more than only one prediction step. Nevertheless also these implementations are not suitable for MMCs with a high number of modules.

Instead of controlling the individual modules’ switching states with the MPC algorithm directly, a modulator can be used. The modulator typically uses a PWM to implement a given reference voltage for the individual arms. Modulators for MMCs can be designed such that all capacitor voltages within one arm are balanced around a mean value and the voltage reference is implemented at the output of the arm with only small errors [15] and low computational burden [16]. With a modulator, the MMC can be modelled by averaged models. Common modelling approaches are shown in Fig. 2. Average modelling means that no actual switching states are considered in the MPC. It is rather assumed that each MMC arm can generate a continuous output voltage, such that traditional MPC algorithms known from non switched systems can be used. As a result, the computational burden of the MPC algorithm is independent of the number of modules and can also be used for MMC systems with hundreds of modules (as e.g. in HVDC).

In [17] a bilinear MMC average model is linearised around the current operation point and a linear MPC algorithm based on a quadratic program (QP) is presented. In [18] the linearised modelling approach from [17] is compared with using a non-linear prediction model for the MPC and it is concluded that the prediction error resulting from the linearisation has a significant influence on the control performance. Nevertheless, the computational burden resulting from the non-linear prediction model is very high, such that a real time implementation of the necessary online optimisation is hardly possible.

In this paper, an MPC algorithm based on the MMC modelling approach shown in Fig. 2(c) is proposed. It will be shown that the resulting MMC model can be easily linearised without a large prediction error to overcome the drawbacks of the MPC schemes proposed in [17] and [18]. The control performance will be evaluated with simulation results and compared to the performance of the cascaded PI control system from [6].

The paper is organized as follows. A linearised MMC model is derived in section II. Section III presents the constraint formulation to be used for the MPC scheme. The MMC model and the constraint formulation are combined in an optimisation problem in section IV resulting in the MPC law. Finally, in section V simulation results are shown and discussed before concluding in section VI.

II. MMC MODELLING

As derived e.g. in [6], the MMC’s current dynamics can be described in decoupled DC side (marked with ‘e’ in the following) and AC side components (marked with ‘a’). Both components have a three phase characteristic such that they can be transformed to the αβ-frame using the Clarke-transformation. The equivalent circuit(s) of these decoupled components is shown in Fig. 3. Note, that the AC side current i_{1u,α,β} has no 0-component due to the open star point on the grid side. Nevertheless, the AC side voltage v_{a,α,β} can have one
The energy dynamics in the individual arms can be found by the product of the arm output voltage $v_{xy}$ and the arm current $i_{xy}$, such that

$$\frac{dw}{dt} = K^{-1}_{v}\cdot \begin{bmatrix} -v_{a,\alpha,30} + 1/2 v_{e,\alpha,30} \\ v_{a,\alpha,30} + 1/2 v_{e,\alpha,30} \end{bmatrix} + K^{-1}_{i}\cdot \begin{bmatrix} 1/2 i_{k,\alpha,30} + i_{\phi,\alpha,30} \\ -1/2 i_{k,\alpha,30} + i_{\phi,\alpha,30} \end{bmatrix},$$

(5)

where $w = [w_{1u}, w_{2u}, w_{3u}, w_{1l}, w_{2l}, w_{3l}]^T$ is the vector of arm energies in abc coordinates, $K_{v,\alpha,30}$ is the Clarke transformation matrix and $i_{k,\alpha,30} = [i_{k,\alpha,30}]^T$.

For the energy dynamics one can now neglect the part $v_{a,\alpha,30}$ and $v_{e,\alpha,30}$ small compared to the grid and DC voltage values. For the following it is assumed

$$[v_{a,\alpha} v_{a,\beta} v_{0,\alpha}]^T \approx [v_{e,\alpha} v_{e,\beta} v_{0,\alpha}]^T = v_{dc,\alpha,30}$$

(6)

(7)

Evaluating (5) with (6) and (7) enables to derive the linearised energy dynamics:

$$\frac{dw}{dt} = \begin{bmatrix} K_{v,\alpha,30}^{-1} \cdot (-v_{e,\alpha,30} + 1/2 v_{e,\alpha,30}) \\ K_{v,\alpha,30}^{-1} \cdot (v_{e,\alpha,30} + 1/2 v_{e,\alpha,30}) \end{bmatrix} + \begin{bmatrix} K_{i,\alpha,30}^{-1} \cdot (1/2 i_{k,\alpha,30} + i_{\phi,\alpha,30}) \\ K_{i,\alpha,30}^{-1} \cdot (-1/2 i_{k,\alpha,30} + i_{\phi,\alpha,30}) \end{bmatrix} = A_w (v_{\phi,\alpha,30}) \cdot \begin{bmatrix} i_{k,\alpha,30} \\ i_{\phi,30} \end{bmatrix}.$$

Note that the grid voltage $v_{dc,\alpha,30}$ is changing over time and thus $A_w$ is dependant on the grid angle $v_{\phi,\alpha,30}$, Nevertheless this change can be predicted throughout the grid period, such that $A_w$ is known a priori. Finally the MMC can be modelled with

$$\frac{d}{dt} \begin{bmatrix} i_{k,\alpha,30} \\ i_{\phi,\alpha,30} \end{bmatrix} = \begin{bmatrix} A_e & 0 \\ 0 & A_a \end{bmatrix} \begin{bmatrix} i_{k,\alpha,30} \\ i_{\phi,\alpha,30} \end{bmatrix} + \begin{bmatrix} B_e & 0 \\ 0 & B_a \end{bmatrix} \begin{bmatrix} v_{dc,\alpha,30} \\ v_{\phi,\alpha,30} \end{bmatrix},$$

(9)

where $0$ is a matrix of zeros. For using the model as a MPC prediction model, it is discretised using the exact zero order hold (ZOH) discretisation method. Note, that due to the time variance of $A_w$, the model needs to be discretised for each and every grid angle $v_{\phi,\alpha,30} \in \{\pi/n \mid n \in \mathbb{N}, 1 \leq n \leq 2\pi/\omega_T\}$.

With ZOH, it is assumed that $A_w(v_{\phi,\alpha,30})$ is constant over the sampling interval $T_s$. In the real world, this is not true, as $v_{\phi,\alpha,30}$ and therefore $v_{\phi,\alpha}$ are continuously changing throughout the sampling interval. To avoid modelling errors, an effective grid voltage $v_{\phi,\alpha,eff}$ is introduced, representing the average value of the grid voltage over sampling period $k$, such that

$$v_{\phi,\alpha,eff} = \int_{kT_s}^{(k+1)T_s} \frac{v_{\phi,\alpha}(t)}{T_s} dt = \int_0^{T_s} \frac{V_g}{T_s} \cos(\varphi_{g,k} + \omega g t) dt$$

(10)
is used within (8) to find \( A_n \) before applying the standard ZOH discretisation for each \( \varphi_k \) throughout the grid period on (9).

### III. Constraints Formulation and Approximation

The physical constraints in an MMC are the maximum arm current \( i_{\text{arm,max}} \) and grid current \( i_{\text{grid,max}} \), as well as the maximum module voltage and therefore the arm energy value \( C/2N \cdot (N \cdot v_{\text{C,max}})^2 \). Based on the previously derived model, the constraints can be formulated as

\[
-i_{\text{arm,max}} \leq \left[ K_{0}^{-1} \cdot \left( i_{\alpha} + 1/2 \cdot i_{\alpha,0} \right) \right] \leq +i_{\text{arm,max}},
\]

(11)

\[
-i_{\text{grid,max}} \leq \left[ K_{0}^{-1} \cdot \left( i_{\alpha,0} + 1/2 \cdot i_{\alpha} \right) \right] \leq +i_{\text{grid,max}},
\]

(12)

\[
0 \leq w \leq C \frac{NC}{2N} \cdot (N \cdot v_{\text{C,max}})^2 = \frac{NC}{2} \cdot v_{\text{C,max}},
\]

(13)

Additionally, the output voltages of the individual MMC arms are limited due to the available inner arm voltage \( v_{\Sigma}^a \) or arm energy \( w_{xy} \), respectively, as it cannot be higher than \( v_{\Sigma}^a \).

Therefore, the control input is constrained by the energy states in the previously derived model:

\[
0 \leq \left[ K_{0}^{-1} \cdot \left( -v_{\alpha,0} + 1/2 \cdot v_{\alpha,0} \right) \right] \leq \sqrt{\frac{2N}{C} \cdot w} \quad (14)
\]

The square root on the right hand side of (14) represents a non-linearity, which can be approximated as follows. The approximation has to be valid in a certain range of \( w_{xy} \) to cover the whole reasonable operating range of the MMC. The upper limit of this range is given by the module voltage constraint (13). The lower limit is chosen somewhat lower (30%) than the lowest energy level \( W_{\text{min}} \) expected in the steady state, such that the approximation range is

\[
0.7 \cdot W_{\text{min}} \leq w_{xy} \leq \frac{NC}{2} \cdot v_{\text{C,max}}^2
\]

where \( W_{\text{min}} \) can be evaluated based on the equations for the energy reference presented in Sec. IV-A. As shown in Fig. 4, the actual constraint can be approximated by a number of straight lines. Each line \( v_{\text{m}}^a \) is given as \( v_{\text{m}}^a = a_m \cdot w + b_m \) resulting in a linear expression for the square root. Of course this approximation also results in an error. As shown in Fig. 4(d), the error decreases with an increasing number of approximation lines. The approximated constraint for the output voltage of arm \( xy \) is written as

\[
v_{xy} \leq a_1 \cdot w_{xy} + b_1, \quad \ldots, \quad v_{xy} \leq a_M \cdot w_{xy} + b_M
\]

(15)

for \( M \) approximation lines, resulting in an arbitrarily accurate approximation for large \( M \). Note, that each line represents six additional linear constraints (the MMC has six arms) when it comes to the formulation of the optimisation problem, such that one should not exceed the necessary number of approximation lines.

### IV. MPC Formulation

Based on the linear MMC model and the linear representation of the MMC’s system constraints, an MPC law formulation is presented in this section. First, equations for the reference values of the MMC are derived in Sec. IV-A before the resulting linear MPC law based on a quadratic program is presented in Sec. IV-B.
TABLE II
MPC WEIGHTING PARAMETERS

<table>
<thead>
<tr>
<th></th>
<th>Fig. 6 (105 µF)</th>
<th>Fig. 7 (60 µF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_s = 1/T_s )</td>
<td>1.5 kHz</td>
<td>1.5 kHz</td>
</tr>
<tr>
<td>( N_p )</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>( \lambda_{ve,αβ}, \lambda_{ve,0}, \lambda_{ve,αβ0} )</td>
<td>1000</td>
<td>10^4</td>
</tr>
<tr>
<td>( \lambda_{dc} )</td>
<td>10^4</td>
<td>100</td>
</tr>
<tr>
<td>( \lambda_w )</td>
<td>10</td>
<td>5</td>
</tr>
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A. Reference Values

For the MPC formulation, the state (\( r_k \)) and input references (\( u_{ref,k} \)) are essential to achieve the desired control performance because these references guide the MMC to the ideal steady-state trajectory. With a given DC voltage \( V_{dc} \) and grid voltage amplitude \( V_g \) as well as a reference output power \( P_{ref} \), the steady-state references for all state variables of the MMC can be calculated. As a starting point, the continuous current references are induced by power relations as

\[
i_{\alpha,β0}^* = \begin{bmatrix} 0 \\ \frac{I_g^*}{3} \end{bmatrix}, \quad i_{\alpha,β0}^* = K_{\alpha,β0} \begin{bmatrix} I_g^* \cos(\omega g t) \\ I_g^* \cos(\omega g t - 2\pi/3) \end{bmatrix},
\]

where \( \omega_g \) is the grid angular frequency. \( I_g^* = P_{ref}/V_{dc} \) and \( I_g^* = 2S/3V_g \) are the amplitudes of the desired DC and AC side currents.

Also the control inputs have a steady state reference value that is based on the reference currents and the MMC’s impedance values. It is given by

\[
u_{ref} = \begin{bmatrix} v_{\alpha,β0,ref}^\delta \\ v_{α,β0,ref}^\delta \end{bmatrix} \text{ with } \begin{bmatrix} 0 \\ \frac{R_g}{3} \end{bmatrix} \cdot I_{dc}^*
\]

\[
v_{α,β0,ref}^\delta = \begin{bmatrix} 0 \\ \frac{R_g}{3} \end{bmatrix} \cdot I_{dc}^* \quad \text{and} \quad v_{α,β0,ref}^\delta = \begin{bmatrix} \cos(\omega g t + \arg(Z_a)) \\ \cos(\omega g t + \arg(Z_a) - 2\pi/3) \end{bmatrix}
\]

where \( Z_a = \frac{R_g}{2} + \frac{R_g + j\omega_g \left( \frac{R_g}{2} + L_g \right)}{} \) is the complex impedance of the MMC's AC side (cf. Fig. 3(b)).

The continuous energy references can be computed via the equations given in [4]. An example of energy variation for the upper arm of the first phase is

\[
e_{1a} = \frac{P_{ref}}{12 m \omega_g} \left( 4 \sin(\omega g t) - m \sin(2\omega g t) - 2 m^2 \sin(\omega g t) \right)
\]

where \( m = 2V_g/V_{dc} \). The full energy references can be stated as

\[
w^* = \frac{C}{2N} \left( \psi_\Sigma^* \right)^2 + e_{123,al}.
\]

where \( \psi_\Sigma^* \) is the rated inner arm voltage.

As a consequence from the above equations, the state references are obtained by simply sampling the continuous references as

\[
x_{ref,k} = \begin{bmatrix} i_{\alpha,β00}^e(kT_s) \\ i_{α,β0}^e(kT_s) \end{bmatrix} \quad w^*(kT_s)^T,
\]

while the input references have to be compensated in the same manner as done in (10) resulting in

\[
u_{ref,k} = \frac{1}{T_s} \int_{kT_s}^{(k+1)T_s} u_{ref}(t) \, dt.
\]

Even though the state references can be constructed for all current and energy states, only the DC current and energy references are employed when the MMC operates as a rectifier. The energy references already implicitly contain the AC current trajectory in the energy variation equation. In contrast, all components of the input references are used to achieve a good steady-state behaviour. The references are evaluated for the whole prediction horizon \( N_p \) and packed in reference matrices \( X_{ref,k} = [x_{ref,k}, \ldots, x_{ref,k+N_p-1}] \) and \( u_{ref,k} = [u_{ref,k}, \ldots, u_{ref,k+N_p-1}] \).

B. Quadratic Programm (QP)

In the following it will be shown how to formulate the MPC law as a quadratic program (QP). QPs with linear constraints have general properties that make them comparatively simple to solve. There are many (also commercial) solvers available [19]. For solving QPs within the required sampling time (\( T_s \leq 1 \text{ms} \)) FPGAs can be used implementing either online solvers [20] or so called explicit MPC schemes [21]-[23].

The discretisation of (9) with the help of (10) leads to

\[x_{k+1} = A_d(\varphi_{g,k}) \cdot x_k + B_d(\varphi_{g,k}) \cdot u_k,
\]

where \( x_k = [i_{\alpha,β00,k}, i_{\alpha,β,k}, w_k]^T \) is the state vector, \( u_k = [v_{\alpha,β00,k}, v_{α,β0,ref}^\delta]^T \) is the control input vector, and \( A_d(\varphi_{g,k}) \) and \( B_d(\varphi_{g,k}) \) are the time-varying discrete model matrices depending on the grid angle \( \varphi_{g,k} \).

The MPC control law to obtain the optimal control input sequence \( U_k \) can therefore be written as

\[
\min_{U_k} \sum_{l=0}^{N_p-1} \| x_{k+l+1} - x_{ref,k+l} \|^2_Q + \| u_{k+l} - u_{ref,k+l} \|^2_R
\]

(17a)

s.t. \( x_{k+l+1} = A_d(\varphi_{g,k+l}) x_{k+l} + B_d(\varphi_{g,k+l}) u_{k+l} \) \quad (17b)

\[\lambda_{min} \leq K_j x_{k+l+1} \leq \lambda_{max}\]

(17c)

\[G u_{k+l} \leq W + K_2 x_{k+l+1} \quad \forall l \in \{0, \ldots, N_p-1\}\]

(17d)

where \( U_k = [u_{k,1}, \ldots, u_{k+N_p-1}]^T \) is the future input vector, \( N_p \) is the prediction horizon and \( \varphi_{g,k+l} = \varphi_{g,k} + l \cdot T_s \omega_g \) is the predicted grid angle. \( x_{k+l+1}, u_{k+l}, r_{k+l}, \) and \( u_{ref,k+l} \) denote the \( l \)-th step prediction states, inputs, state reference and input reference, where \( x_k \) is the current state value.

The quadratic cost function consists of a state reference tracking and input reference tracking term, where \( \| z \|_P \) denotes a 2-norm with the weighting matrix \( P \). The weighting factors are defined as positive semi-definite diagonal matrices of

\[Q = \text{diag} \left[ 0_2 \lambda_{dc} \ 0_2 \lambda_{w} \ 1_6 \right] \geq 0,
\]

\[R = \text{diag} \left[ \lambda_{ve,αβ} \cdot 1_2 \lambda_{ve,0} \lambda_{ve,αβ0} \cdot 1_3 \right] \geq 0,
\]

where \( 0_m \) represents a vector of zeros with dimension \( m \). The constant constraints such as maximum arm currents, grid
In this section, comprehensive simulation results of the proposed MPC scheme are presented. To give some context to the performance, the same MMC parameters as in [18] are used. The parameters correspond to a MVDC source to be implemented at ETH Zürich [24]. The MPC scheme has been implemented as a function in Matlab/Simulink using the YALMIP toolbox [25]. To reduce the simulation time, an average MMC model (cf. [26], [27]) has been used for the simulation.

The results are shown in the left part of Fig. 6. The MPC parameters for this scenario are noted in the first column of Tab. II. It can be seen that the DC current tracking is extraordinary fast in the transients and very precise in the steady state. At the same time, the energy balancing successfully avoids that the inner arm voltages exceed their maximum value, even though this value is very close to the maximum voltage necessary for the steady state trajectory of the arm energies. The arm energies are balanced with circulating currents that occur after the reference steps and disappear in the steady state when the arm energies are at their reference value again. Nevertheless, the power reversal happens almost twice as fast as with the PI controller. Note, that the circulating currents are comparably small even during the transients. This is due to the low amount of energy stored in the MMC, such that only little current is needed to balance the energy.

VI. CONCLUSION

This paper presents a novel MPC scheme based on an approximated linear average prediction model for grid connected MMCs using a modulator (e.g., PWM) to determine the switching signals for the individual MMC modules. Thus, the computational burden is independent of the number of MMC modules. Simulation results show the superior transient performance of the proposed linear MPC, which is comparable to the nonlinear MPC from [18]. As the MPC in this paper uses a linear prediction model in combination with a novel linearised constraint representation it can be formulated as a convex quadratic program (QP). This is much simpler to implement in real time than the nonlinear MPC presented in [18]. A comparison to a classical cascaded PI control scheme is used to prove the importance of considering the MMC's voltage and current constraints within the control system. A second simulation scenario shows that the proposed MPC can be used to operate MMC designs with very low control/voltage margin and therefore very small module capacitance values. This can lead to compact and relatively cheap MMC realisations (capacitance value reduction compared to cascaded PI of more than 40% in this paper), while keeping a fast transient behaviour (MPC twice as fast as cascaded PI).
Fig. 6. Simulation results with the proposed MPC and the cascaded PI control scheme from [6] (model based variant) with a module capacitance of 105 μF. Note that the time scale is different in both plots and that the sampling frequency of the PI control has been set to 15 kHz.
Fig. 7. Simulation results with the proposed MPC for a module capacitance of only 60 µF. Note the large voltage ripple of the inner arm voltages shown in the lowest plot is ranging from the arm output voltage to the maximum inner arm voltage even in steady state. A reasonable operation of this MMC using the cascaded PI control scheme is not possible.

REFERENCES


