Core Losses under DC Bias Condition based on Steinmetz Parameters

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Abstract—When designing inductive components, calculating core losses is a difficult and not yet entirely solved problem. In particular it is impossible to predict the influence of a DC premagnetization on the losses without performing extensive measurements. For this work, different materials have been tested to gain information how core losses are influenced by a premagnetization. Measurements on molypermalloy powder, silicon steel, nanocrystalline material, and ferrite cores have been performed. Of the tested materials, a premagnetization mainly influences losses in ferrites and nanocrystalline materials, whereas the influence of a premagnetization in molypermalloy powder cores, and cores of silicon steel is negligible. The Steinmetz Premagnetization Graph (SPG) that shows the dependency of the Steinmetz parameters (α, β and k) on premagnetization is introduced. This permits the calculation of core losses under DC bias conditions. Such graphs are given for different materials and different operating temperatures. In addition, a detailed description of the test system is given, as high accuracy is crucial.

Index Terms—Core losses, ferrite, Steinmetz, DC bias.

I. INTRODUCTION

When designing power electronic systems, predicting core losses is a difficult, and not yet entirely solved problem. Particularly, the influence of a DC bias on the losses is not entirely clarified. The most used equation that characterizes core losses is the power equation [1]

\[ P_v = k f^\alpha \dot{B}^\beta \]  

(1)

where \( \dot{B} \) is the peak induction of a sinusoidal excitation with frequency \( f \), \( P_v \) is the time-average power loss per unit volume, and \( k, \alpha, \beta \) are material parameters. The equation is often referred to as the Steinmetz equation, named after Charles P. Steinmetz, who proposed a similar equation, without the frequency dependence, in 1892 [2]. The material parameters \( k, \alpha, \beta \) are accordingly referred to as the Steinmetz parameters. They are valid for a limited frequency and flux density range. Unfortunately, the Steinmetz equation is only valid for sinusoidal excitation. This is a huge drawback, because in power electronics applications the material is mostly exposed to nonsinusoidal flux waveforms.

To overcome this limitation, and determine losses for a wider variety of waveforms, different approaches have been developed. A common method to determine core losses is to break up the loss into hysteresis loss \( P_h \), classical eddy current loss \( P_{cl} \), and "excess loss" \( P_{exc} \) [3], [4]

\[ P_v = P_h + P_{cl} + P_{exc} \]  

(2)

The approach of loss separation has a practical disadvantage: such models are based on parameters, which are not always available and difficult to extract. Hence, simpler models are desired.

Another approach to overcome the limitation of the Steinmetz equation is derived in [5], [6], and [7]. The analysis in [5] is motivated by the fact that the loss due to domain wall motion has a direct dependency of \( dB/dt \). As a result, a modified Steinmetz equation is proposed. In [6] the approach is further improved and in [7] a method how to deal with minor hysteresis loops is presented and some minor changes on the equation are made. The approach of [5], [6], and [7] leads to the improved Generalized Steinmetz Equation (iGSE)

\[ P_v = \frac{1}{T} \int_0^T k_1 \frac{dB}{dt} |(\Delta B)^{\beta-\alpha} dt \]  

(3)

where \( \Delta B \) is peak-to-peak flux density and

\[ k_1 = \frac{k}{(2\pi)^{\alpha-1} \int_0^{2\pi} |\cos \theta|^{\alpha-\beta} d\theta}. \]  

(4)

The parameters \( k, \alpha, \beta \) are the same parameters as used in the Steinmetz equation (1). The iGSE is capable of accurately calculating losses of any flux waveform, without requiring extra characterization of material parameters beyond the parameters for the Steinmetz equation. Nevertheless, the iGSE has a drawback: it neglects the fact that core losses vary under DC bias conditions, i.e. the Steinmetz parameters change under DC bias condition.

In many power electronics applications magnetic components are biased with a DC or low-frequency premagnetization, e.g. in Switched-Mode Power Supplies (SMPS). Within SMPS circuits, magnetic components that are operating under DC bias conditions are commonly used, and are often among the largest components. Many publications have shown, that the influence of DC bias on the material properties can not be neglected [8]–[13]. An approach how to handle DC bias losses is described in [14], [15] and [16]. There, losses are calculated with a loss map that is based on measurements. This loss map stores the loss information for many operating points, each described by the flux ripple \( \Delta B \), the frequency \( f \), and a DC bias \( H_{DC} \). It is explained, how this loss map can be used for calculating iron losses of inductors employed in power electronic systems. One parameter in the loss map is the DC premagnetization, thus the loss increase due to DC bias is considered in this approach. However, extensive measurements are necessary to build the loss.
map. In conclusion, any description of core losses under DC bias condition without extensive measurements is an unsolved problem.

This work proposes a new approach how to describe core losses under DC bias condition. A graph that shows the Steinmetz parameters’ (α, β and k) dependency on premagnetization is introduced in Section IV. This enables the calculation of losses via the Steinmetz equation (1) or the iGSE (3) using appropriate Steinmetz parameters. A core loss measurement test setup has been built for this work and is presented in Section II. The accuracy of the system is analyzed in Section III.

II. TEST SETUP TO MEASURE CORE LOSSES

To perform measurements, the best measurement technique has to be selected first. In [17] different methods are compared. The B-H Loop Measurement has been evaluated as the most suitable. Amongst other advantages, this technique offers rapid measurement (compared to other methods, e.g. calorimetric measurement), copper losses are not measured, and a good accuracy. In Section III the accuracy is analyzed in detail. The principles are as follows: two windings are placed around the Core Under Test (CUT). The sense winding (secondary winding) voltage \( v \) is integrated to sense the core flux density \( B \)

\[
B(t) = \frac{1}{N_2 \cdot A_e} \int_0^t v(\tau) \, d\tau
\]

(5)

where \( N_2 \) is the number of sense winding turns and \( A_e \) the effective core cross section of the CUT. The current in the excitation winding (primary winding) is proportional to the magnetic field strengths \( H \)

\[
H(t) = \frac{N_1 \cdot i(t)}{l_e}
\]

(6)

where \( N_1 \) is the number of excitation winding turns and \( l_e \) the effective magnetic path length of the CUT. The loss per unit volume is then the enclosed area of the B-H loop, multiplied by the frequency \( f \)

\[
P_V = f \int H \, dB.
\]

(7)

The selected approach is widely used [7], [15]. The test system consists of an oscilloscope, a power supply, a heating chamber, and a power stage, as illustrated in Fig. 1. It is controlled by a MATLAB program running on Microsoft Windows. In Table I the used equipment is listed. In Fig. 2 a photograph (a) and the simplified schematic (b) of the power stage is shown. In Table II the most important components employed in the power stage are listed. The power stage is capable of a maximal input voltage of 450 V, output current of 25 A and a switching frequency of up to 200 kHz. With the power stage, it is possible to achieve a rectangular voltage shape across the CUT, that leads to a triangular current shape including a DC bias (if desired). This behavior is illustrated in Fig. 3. To control the DC current, the current is sensed by a DC current transducer. A low frequency sinusoidal excitation is also possible, for this an output filter has been designed to achieve a sinusoidal current/voltage shape for frequencies up to 1 kHz.

III. ACCURACY OF THE MEASUREMENT SYSTEM

The different aspects that influence the accuracy of the measurements are given in the following.

A. Phase Shift Error of Voltage and Current Measurement

According to [18], the error due to an inaccurate measurement of the voltage and current phase displacement can be quantified as

\[
E = 100 \cdot \frac{\cos(\zeta + \phi) - \cos \zeta}{\cos \zeta}, \quad (8)
\]
where $E$ is the relative error in % of the measured core losses, $\zeta$ is the actual phase shift between the sense winding output voltage and the excitation winding current, and $\phi$ is the error in the measurement of $\zeta$. Measurements have shown that $\phi$ is linearly depending on the frequency. In other words, $\phi$ originates from a dead time $T_d$ that is independent of the frequency. This dead time $T_d$ can be measured with a rectangular current shape through a low inductance shunt, and with it the dead time can be compensated.

In Section IV measurements with the material ferrite N87 from EPCOS (core part number: B64290L22X87) are presented, therefore a short discussion about phase shift accuracy is given using the example of this core. In Fig. 4 a simplified equivalent circuit of the Core Under Test (CUT) is given. Winding losses and leakage inductance are assumed to be negligible. The reactance $X_m$ can be calculated as

$$X_m = \omega A_L N_1^2,$$

where $A_L$ is the inductance factor, $N_1$ is the number of primary winding turns, and $\omega = 2\pi f$ is the angular frequency. Hence, for the CUT ($A_L = 2560 \text{nH}$ [19], $N_1 = 10$) and a frequency $f = 100 \text{kHz}$ the reactance is $X_m = 160.8 \Omega$. According to the material datasheet [19], at the operating point $\Delta B = 100 \text{mT}$, $f = 100 \text{kHz}$, and $T = 40^\circ \text{C}$ losses of $P_{\text{Loss}} = 0.2 \text{W}$ are expected. With this information, the equivalent resistor $R_{\text{FE}}$ which represents the core losses can be calculated

$$R_{\text{FE}} = \frac{V_{\text{rms}}^2}{P_{\text{Loss}}} = \left(\frac{N_1 A_{\text{core}} \Delta B}{2\sqrt{2}}\right)^2,$$

where $A_{\text{core}}$ is the equivalent core cross section. For the CUT and operating point, the resistor $R_{\text{FE}}$ is $2.26 \text{k}\Omega$. Now, the angle $\zeta$ can be calculated as

$$\zeta = \arctan \frac{R_{\text{FE}}}{X_m} = 85.9^\circ$$

An uncompensated dead time $T_d$ would result in a phase shift error of voltage and current measurement of $\phi = f \cdot T_d \cdot 360^\circ$.

Inserting (12) in (8), and solving for $T_d$, leads to a tolerable uncompensated dead time for a desired accuracy, e.g. for an accuracy of $\pm3\%$, an uncompensated dead time of $\pm3.5 \text{ns}$ at $100 \text{kHz}$ and $\zeta = 85.9^\circ$ would be tolerable. Measurements have shown, that the dead time compensation leads to lower residual dead times, although a quantification is difficult. Assuming a realistic dead time compensation to an accuracy of $\pm1.5 \text{ns}$, and expecting a system accuracy (only phase shift consideration) of $\pm4\%$, measurements of materials up to an angle of $\zeta = 88.7^\circ$ (at $f = 100 \text{kHz}$) can be performed. At lower frequency measurements the permitted angle $\zeta$ increases for the same accuracy constraint, e.g. at $20 \text{kHz}$, for an accuracy of $\pm4\%$, measurements up to an angle of $\zeta = 89.7^\circ$ are permitted. All measurements presented in the next sections are within this range.

The system has one drawback related to the phase shift: measuring gapped cores (or low permeability cores) is difficult because the angle $\zeta$ substantially increases in this situation. A detailed analysis together with a new method of how gapped cores could be measured is introduced in [13]. However, for this work only un-gapped cores have been measured.

### B. Equipment Accuracy

A typical magnitude/frequency characteristic of the current probe has been provided by the current probe manufacturer LeCroy, from which an AC accuracy of $3\%$ could be extracted. Together with the accuracy of
the passive probe (attenuation accuracy of 1%), and the accuracy of the oscilloscope itself (1.5%, that originates amongst others from the limited vertical resolution of 8 bit), an equipment accuracy of \( \leq |\pm 5.6\%| \) is derived.

C. Capacitive Couplings

Capacitive currents may result in errors and must therefore be avoided. The typical capacitances that are present in windings are

- capacitance between the primary and secondary winding (inter capacitance),
- self capacitance between turns of a winding (intra capacitance),
- and capacitance between the windings and the magnetic core.

According to [20], the inter and intra capacitances increase when the core is grounded, thus the core should not be grounded. Generally, the inter and intra capacitances increase with increasing area between the windings and decrease with distance between the windings. To decrease the inter capacitance, separating the windings is favorable, although separating the windings avoids an absolute uniform winding distribution around the core (ideally, the primary winding should be distributed uniformly around the core to achieve a homogenous flux density distribution).

Another important aspect of the winding arrangement is the chosen number of turns of the primary winding. Even with the use of favorable winding layout, some ringing in current and voltage is inevitable. Less turns are more favorable for two reasons: this additionally decreases parasitic capacitances, and, because the current for the same magnetic operating point is higher, capacitive currents are relatively lower compared to (desired) inductive currents.

D. Temperature

An important aspect for performing the measurement is that the temperature of the CUT is defined and constant. To keep the temperature constant, the test system performs the measurement automatically (starts excitation, controls current, regulates flux (\( \Delta B \)), triggers the oscilloscope, reads values). With such an automated measurement system, a working point is rapidly measured and the losses do not heat the core in the short measurement period.

E. Comparative Measurement and Conclusion

Comparative measurements with the power analyzer Norma D6100 have been performed to confirm the accuracy. The power analyzer is connected to measure the excitation winding current and the sense winding voltage to obtain core losses [17]. The results in the performed working points matched very well. The deviation between the results of the test system and of the power analyzer was always \( \leq |\pm 4\%| \) (up to 100 kHz).

From the equipment accuracy (\( \leq |\pm 5.6\%| \)) and the phase shift accuracy (\( \leq |\pm 4\%| \)), a system accuracy of \( \leq |\pm 9.8\%| \) is calculated. However, based on the results of the comparative measurements, it can be said that the reached accuracy is higher.

Concluding, a test system has been built up, that performs the measurements quickly and leads to sufficiently accurate results.

IV. Core Losses of Ferrites Under DC Bias Condition

In this section, measurement results are presented and a new approach to describe core losses under DC bias conditions is introduced, which is based on a graph that shows the dependency of the Steinmetz parameters (\( \alpha \), \( \beta \) and \( k \)) on premagnetization. This is done using the example of the material ferrite N87 from EPCOS (core part number B64290L22X87 [19]). In Fig. 5 the core losses and in Fig. 6 the core losses normalized to the losses \( P_0 \) at zero premagnetization are shown for different DC bias values. In Fig. 7 the losses are plotted in function of the frequency \( f \), with and without DC bias. Describing losses via the Steinmetz equation (1) or the iGSE (3) is the most common method, hence improving this method would be most beneficial for design engineers. As the iGSE (3) is more suitable for describing core losses in power electronic applications, in all following considerations, the three discussed parameters are \( \alpha \), \( \beta \) and \( k_i \) of the iGSE (\( \alpha \), \( \beta \) are the same as in (1), while \( k_i \) is described in (4)). For the applied waveform as illustrated in Fig. 3 (triangular current/flux shape), (3) leads to

\[
P_v = k_i (2f)^\alpha \Delta B^\beta. \tag{13}
\]

When a core is under DC bias condition, the losses still can be described with the Steinmetz equation (1) or the iGSE (3), i.e. the losses still follow the power equation stated by Steinmetz. However, the Steinmetz parameters must be adjusted according to the DC bias present. As will be shown in the following, a DC bias causes changes in the Steinmetz parameters \( \beta \) and \( k_i \), but not in the parameter \( \alpha \).

- The losses change when keeping \( \Delta B \) and frequency \( f \) constant and varying the DC bias \( H_{DC} \) (cf. Fig. 5). Thus, the Steinmetz parameter \( k_i \) depends on the DC bias \( H_{DC} \) (\( k_i = f(H_{DC}) \)).

![Fig. 5. Core losses under DC bias conditions (ferrite N87, measured on R42 core), \( f = 100 \text{ kHz}, T = 40 \text{ °C} \).](attachment:image.png)
Keeping the frequency \( f \) constant, the factor by which the losses increase due to a premagnetization \( H_{\text{DC}} \) differs for different \( \Delta B \) (cf. Fig. 6). Thus, the Steinmetz parameter \( \beta \) depends on the premagnetization \( H_{\text{DC}} \) as well (\( \beta = f(H_{\text{DC}}) \)).

According to [8], the influence of a DC bias does not depend on the measurement frequency \( f \). This has been confirmed. As can be seen in Fig. 7, at a constant \( \Delta B \), the factor by which the losses increase due to a premagnetization \( H_{\text{DC}} \) is the same for different frequencies \( f \) (the slopes of the curves remain the same). Hence, the Steinmetz parameter \( \alpha \) is independent of the premagnetization \( H_{\text{DC}} \) (\( \alpha = \text{const.} \)). The fact, that \( \alpha \) is constant has been confirmed to frequencies up to 100 kHz, no measurements above this frequency have been performed, hence no information can be given, whether and up to which frequency \( \alpha \) is constant.

For the material N87 from EPCOS the dependencies \( \beta = f(H_{\text{DC}}) \) and \( k_1 = f(H_{\text{DC}}) \) are given in Fig. 8, and normalized to \( \beta_0 \) and \( k_{10} \) in Fig. 9. \( \beta_0 \) and \( k_{10} \) are the Steinmetz parameters at zero premagnetization. We call the graph illustrated in Fig. 9 the Steinmetz Premagnetization Graph (SPG). Its derivation is discussed in Appendix A. The SPG is very useful and it would be valuable to have such a graph in the data sheet of a magnetic material as it would then be possible to calculate core losses under a DC bias condition. Fig. 10 shows how the measured and, based on the SPG, calculated curves compare. For the considered working points (\( \Delta B = 50 \text{ mT}/100 \text{ mT}/150 \text{ mT}, f = 20 \text{ kHz}/100 \text{ kHz} \)) an error of \( \leq 10\% \) has been achieved. In Appendix B SPGs of other materials (Ferroxcube 3F3 (ferrite), EPCOS N27 (ferrite), and VAC VITROPERM 500F (nanocrystalline material)) are given. Furthermore, a discussion how to extract the Steinmetz parameter value \( k \) from the SPG is given in Appendix C.

V. INFLUENCE OF TEMPERATURE

For an accurate core calculation, the temperature is another important parameter that considerably influences core losses. In Fig. 11 the losses normalized to losses \( P_0 \) at zero premagnetization are given for different temperatures. As can be seen, for the material ferrite N87, at higher temperatures the influence of a premagnetization to core losses reduces. The temperature influence is described by extending the SPG to curves of different operating temperatures, as is shown in Fig. 9.
(a) = 20kHz (measured) 
(b) = 100kHz (measured) 
(c) = 100kHz (measured) 
(d) = 20kHz (measured) 
(e) = 100kHz (measured) 
(f) = 150kHz (measured) 

VI. EXAMPLE HOW TO USE THE SPG

In the previous sections, the SPG has been introduced. This section presents now an easy to follow example that illustrates how to calculate core losses of the inductor of a power electronics converter with help of the SPG. In Fig. 12 the schematic and the inductor current waveform of a buck converter, and in Table III the corresponding specifications are given. For the inductor L a DC bias of $H_{DC} = 44\, \text{A/m}$, and a flux density ripple of $\Delta B = 73\, \text{mT}$ is calculated. The following steps lead to the core losses occurring in the inductor:

- For the used material, the corresponding Steinmetz parameters are extracted from the datasheet. This is done by solving (1) at three operating points for $\alpha$, $\beta$, and $k$: $\alpha = 1.25$, $\beta = 2.46$, $k = 15.9$ (values for temperature $T = 40\, ^\circ\text{C}$, at zero premagnetization).
- Next, $k_t$ is calculated according to (4): $k_t = 1.17$.
- $k_t$ and $\beta$ are now adjusted according to the SPG of the material N87 (cf. Fig. 9) for an operating point with $H_{DC} = 44\, \text{A/m}$: $k'_t = 2.8 \cdot k_t = 3.28$ and $\beta' = 1.04 \cdot \beta = 2.56$.
- Now, the losses are calculated according to (3). For piecewise linear waveforms, as is the case in the presented example, the integral of (3) may be split into one piece for each linear segment, so that a complicated numerical integration is avoided [7]. The losses follow as

$$P = V_e \frac{k'(\Delta B)_{\beta'-\alpha}}{T} \cdot \left( \frac{\Delta B}{DT} \right)^\alpha \left( 1 - D \right)^T + \frac{\Delta B}{(1-D)T} \left( 1 - D \right)^T$$

$$= V_e \frac{k'_t(\Delta B)_{\beta' - \alpha}}{T} \cdot \left( \frac{V_{in} - V_{out}}{NA_e} \right)^\alpha DT + \frac{-V_{out}}{NA_e} \left( 1 - D \right)^T$$

$$= 52.8 \, \text{mW}, \quad (14)$$

where $V_e = 3079\, \text{mm}^3$ is the effective core volume, $A_e = 51.26\, \text{mm}^2$ is the effective core cross section, $T = 1/f$ is the period length, and $D = 0.5$ is the duty cycle.

Assuming that the Steinmetz parameters had not been adjusted according to the SPG in the example above, the losses would have been calculated as $P = 24.5\, \text{mW}$, which is an underestimation by a factor of more than two.

VII. CORE LOSSES UNDER DC BIAS CONDITION OF DIFFERENT MATERIALS

Different materials have been tested to gain information how core losses are influenced by a premagnetization in different materials. Measurements on a molypermalloy powder core (Magnetics MPP 300u) and cores of silicon steel (tested: ET165-35 grain-oriented steel with lamination thickness 0.35 mm, M470-50A non-oriented...
steel with lamination thickness 0.5 mm) have shown that the losses are unaffected (or change only very little) by a premagnetization. The measurements on the silicon steel cores have been performed up to a DC magnetic field strengths of 1400 A/m (core is already out of its linear region). For very high $H_{DC}$, a small loss increase has been observed (e.g. for steel M470-50A 25% higher losses than at zero premagnetization have been observed at a DC magnetic field strengths of 1400 A/m). The tested powder core (Magnetics MPP 300u; part number: C055433A2) has been tested up to a DC magnetic field strengths of 1200 A/m, up to that operating point the loss change is negligible small.

![Fig. 13. Core losses under DC bias conditions; material VITROPERM 500F (VAC); core: W452; $f = 100$ kHz, $T = 40 \degree C$.](image)

**TABLE IV**

<table>
<thead>
<tr>
<th>Material Class</th>
<th>Measured Material(s)</th>
<th>Impact on Losses?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soft Ferrite</td>
<td>EPCOS N87, N27, T35</td>
<td>yes</td>
</tr>
<tr>
<td>Nanocrystalline</td>
<td>VITROPERM 500F (VAC)</td>
<td>yes</td>
</tr>
<tr>
<td>Silicon Steel</td>
<td>M470-50A non-oriented steel</td>
<td>negligible</td>
</tr>
<tr>
<td>Molypermalloy Powder</td>
<td>Magnetics MPP300</td>
<td>negligible</td>
</tr>
</tbody>
</table>

Losses in the nanocrystalline material VITROPERM 500F from Vacuumschmelze increase under DC bias condition, as can be seen in Fig. 13. The SPGs of the material VITROPERM 500F and of some more ferrites are given in Appendix B. In Table IV an overview of the tested materials is given. Tests have been performed only on the above listed components, hence, a general declaration of the whole material class cannot be made with 100% certainty.

**VIII. CONCLUSION AND FUTURE WORK**

A graph that shows the dependency of the Steinmetz parameters ($\alpha$, $\beta$ and $k$) on premagnetization, i.e. the Steinmetz Premagnetization Graph (SPG) has been introduced. Based on the SPG, calculating core losses under DC bias condition becomes possible. It is shown, that the graph is independent of the frequency $f$, and therewith applicable for a wide frequency range. This new approach of describing losses under DC bias condition is promising due to its simplicity; only the SPG would have to be added to a magnetic material data sheet.

**APPENDIX A**

**DERIVATION OF THE STEINMETZ PREMAGNETIZATION GRAPH**

The Steinmetz parameters in function of $H_{DC}$ are described with a fourth order series expansion

$$
\begin{bmatrix}
\alpha \\
\beta \\
\kappa_i
\end{bmatrix}
= 
\begin{bmatrix}
\alpha_0 & 0 & 0 & 0 & 0 \\
\beta_0 & p_{\beta 1} & p_{\beta 2} & p_{\beta 3} & p_{\beta 4} \\
\kappa_{i0} & p_{\kappa i1} & p_{\kappa i2} & p_{\kappa i3} & p_{\kappa i4}
\end{bmatrix}
\cdot
\begin{bmatrix}
1 \\
H_{DC} \\
H_{DC}^2 \\
H_{DC}^3 \\
H_{DC}^4
\end{bmatrix}
(15)
$$

or

$$
S = P \cdot H.
(16)
$$

To extract the dependency of the Steinmetz parameters on the premagnetization, one has to find the right coefficients of the matrix $P$. This is an optimization problem. A least square algorithm has been implemented that fits measured curves with calculated data by minimizing the relative error at 3 different values of $\Delta B$, each measured at two frequencies, and 6 premagnetization values $H_{DC}$ (including $H_{DC} = 0$). The losses are calculated according to (13) with Steinmetz parameters from (15)/(16). In the initial matrix $P$, all elements $p_{\kappa}$ (cf. (15)) are set to zero. The values that represent the Steinmetz values under no DC bias condition ($\alpha_0$, $\beta_0$, and $\kappa_{i0}$) have reasonable initial values. As an optimization constraint, it is assumed that $\alpha(H_{DC}) > 1$ and $\beta(H_{DC}) > 2$ for all values of $H_{DC}$. The optimization is based on the MATLAB function fminsearch() that applies the Downhill-Simpex-Approach of Nelder and Mead [21]. This optimization procedure leads to graphs for the Steinmetz dependency as Fig. 8, or normalized to $\beta_0$ respectively $k_{i0}$ to the SPG as e.g. shown in Fig. 9.

**APPENDIX B**

**SPGS OF OTHER MATERIALS**

In Fig. 14 the SPG is given for the material EPCOS N27, and in Fig. 15 for the material Ferroxcube 3F3. In Fig. 16 the SPG for the nanocrystalline material VITROPERM 500F from Vacuumschmelze (VAC) is depicted. The independency of the frequency has been confirmed for all materials ($\alpha = \text{constant}$).

**APPENDIX C**

**CLASSIC STEINMETZ PARAMETER $k$**

A short discussion how to extract the Steinmetz parameter value $k$ (not $k_0$) from the SPG is given in the following. According to (4), for $k_0$ we have

$$
k = \frac{k_{i0}(2\pi)^{\alpha-1}\int_0^{2\pi} |\cos \theta|^{\alpha-2-\alpha} d\theta}{\kappa_{i0}(2\pi)^{\alpha-1}\int_0^{2\pi} |\cos \theta|^{\alpha-2-\alpha} d\theta}.
(17)
$$
that is, assuming $\alpha = \text{constant}$,

$$k = k_0 \frac{2^\beta}{k_0} = \frac{k_0}{k_0}, 2 \left( \frac{\beta}{\beta_0} - 1 \right) \beta_0,$$  

where $\frac{\beta}{\beta_0}$ can be extracted from the SPG. Of course, it is conceivable to write $k/k_0$ in the SPG, instead of $k_1/k_0$. However, because the built test system excites the core with a triangular current shape, $k_1/k_0$ has been chosen for the graph. The iGSE is in any case very broadly used, hence, to avoid further calculations, directly having information about $k_1$ is often desired.

REFERENCES