Leakage Inductance Modelling of Transformers: Accurate and Fast Models to Scale the Leakage Inductance Per Unit Length

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Accurate and Fast Models to Scale the Leakage Inductance Per Unit Length

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Abstract
Fast and accurate transformer leakage inductance models are crucial for optimisation-based design of galvanically isolated converters. Analytical models are rapidly executable and therefore specially suitable for such optimisations. These analytical leakage inductance models typically consist of two steps: First, acquire the leakage inductance per unit length and second, scale this value with a suitable length. In this paper, the term leakage length is introduced for the scaling length. It is shown that the leakage length depends on the magnetic energy distribution and the most influential factors are determined. Furthermore, two accurate and fast leakage length models for E-core and U-core transformers with concentric windings are proposed: The Empirically Corrected Axial Flux (ECAF) model is based on a compact modification of the known axial flux formula. The cut line (CL) model pursues a semi-analytical approach and achieves high accuracy at the cost of higher computational effort. The models are verified with more than 6000 FEM simulations and the error of both models is significantly lower than the error of the known axial flux formula.

1 Introduction
Leakage inductance is an important property of transformers in galvanically isolated power electronic converters as it significantly influences the operation of the converter. Galvanically isolated converters are key components in several applications that enable a more sustainable energy system such as photovoltaic inverters [1], electric traction systems [2], and power quality control of the electric power grid [3].

In the design stage of such converters, the operating and design parameters are usually determined in an optimisation procedure, before the components are physically built [2, 4]. During this optimisation procedure, the parameters are recalculated several thousand times (see e.g. [4]). Consequently, the employed models have to be accurate and fast to deliver accurate results within a reasonable amount of time. Analytical models are very well suited for this purpose as they are more compact and therefore faster than reluctance network modelling and numerical methods such as the finite element method (FEM) [5–7].

Analytical modelling of the transformer leakage inductance typically consists of two steps [8–13]:

1. Calculate the leakage inductance per unit length $L'_\sigma$ from a 2D cross section (Fig 1a).
2. Scale the leakage inductance per unit length $L'_\sigma$ to the actual leakage inductance $L_\sigma$ (Fig. 1b).

In previous work [5, 7], we compared leakage inductance per unit length models with respect to accuracy and computational effort. The present work examines the appropriate scaling length. In [14–17], the mean length of turns $MLT$ is used to scale the leakage inductance per unit length. However, using the $MLT$ as scaling length is not exactly accurate as the leakage inductance is deduced from the stored magnetic energy. In case of curved winding sections such as around a circular center leg and at the edges of a rectangular center leg, the magnetic field is curved. Hence, an energy weighted mean length is required to take the curvature of the magnetic energy distribution into account. This energy weighted mean length is referred to as "leakage length" $l_\sigma$ in this paper to avoid confusion with other terms. Hence, the leakage inductance assuming an axisymmetric arrangement as shown in Figs. 1a,b is analytically...
calculated according to

\[
L'_{1\sigma} \rightarrow \text{Fig. 1a} \quad \ast \quad l_{1\sigma} \rightarrow \text{Fig. 1b} = L_{1\sigma}
\]

The leakage inductance models proposed in [8–10, 13] do not give a specific equation to calculate the leakage length at all. Margueron [11, 12] uses the local maximum of the magnetic energy density to calculate the leakage length. This is disadvantageous because the magnetic field needs to be calculated over the whole 2D cross section to obtain the maximum value, which leads to considerable computational effort. In [18], a known formula for the leakage length is given, however, without derivation. A derivation of the known formula can be found in [19]. However, this formula assumes purely axial leakage flux (1D-field) in the transformer window (see Fig. 1c) and is therefore not generally applicable. Hence, a comprehensive treatise and general model of the leakage length has not been published yet.

Therefore, this paper derives the leakage length for curved winding sections based on axisymmetric 2D transformer cross sections. Furthermore, two models of the leakage length are proposed: The first proposed model is called empirically corrected axial flux (ECAF) model (sec. 4.1) and is based on a modification of the known axial flux formula. The second proposed model is called cut line (CL) model (sec. 4.2) and pursues a semi-analytical approach. The proposed models are applicable to E-core and U-core transformers with concentric windings.

The rest of this paper is organised as follows: First, fundamental assumptions are briefly summarised and the typical leakage inductance modelling steps are elucidated in sec. 2. Sec. 3 describes the general concept of the leakage length, the derivation of the axial flux formula, and identifies the main geometric influences on the leakage length. Sec. 4 presents the proposed ECAF and CL models. The models are validated by more than 6000 2D-FEM simulations in sec. 5.

2 Analytical Leakage Inductance Modelling

2.1 Fundamentals

In analytical leakage inductance modelling, the windings are unified to rectangular winding blocks as shown in Figs. 2a&b. The exact winding type such as Litz, round, and foil is usually neglected [5, 8–12]. The magnetomotive forces \( MMF \) of primary and secondary side of the transformer are balanced according to (1) to obtain the leakage field distribution.

\[
MMF_1 = N_1 \cdot I_1 = -MMF_2 = -N_2 \cdot I_2
\]

where \( N \) and \( I \) are the number of turns and the current through the winding. Indices 1 and 2 indicate primary and secondary side of the transformer. Further details on basic principles of leakage inductance
calculations can be found in standard literature (e.g. [20, 21]).

2.2 Modelling Steps

Recent modelling approaches pursue a "Double-2D" concept [5, 6, 22–25]. This means that the calculation is based on two cross sections of the transformer: inside the core window (IW) and outside the core window (OW) as depicted in Figs. 2a&b. This approach is more accurate than "Single-2D" modelling [5] and is therefore also used in this paper. Double-2D modelling consists of two steps:

1. The leakage inductance per unit length is calculated for both, IW and OW cross section with an appropriate analytical model, resulting in $L'_{\text{g, in}}$ and $L'_{\text{g, out}}$. See [5] for a comparison of different leakage inductance per unit length models with respect to accuracy and computational effort. The assumed geometries imply straight windings of infinite length.

2. The IW and OW leakage inductance per unit lengths are scaled with their respective partial leakage length $l_{p,\text{in}}$ and $l_{p,\text{out}}$ as shown in Figs. 2e–h. The total leakage inductance according to the Double-2D approach is then obtained with (2).

$$L_{\text{g}} = L'_{\text{g, in}} \cdot l_{p,\text{in}} + L'_{\text{g, out}} \cdot l_{p,\text{out}}$$

The partial leakage lengths $l_{p,\text{in}}$ and $l_{p,\text{out}}$ depend on the core shape and can be derived from the IW and OW leakage lengths $l_{\sigma,\text{in}}$ and $l_{\sigma,\text{out}}$. The IW and OW leakage lengths $l_{\sigma,\text{in}}$ and $l_{\sigma,\text{out}}$ refer to the rotation of the particular cross section and are visualised in Figs. 2c&d.

2.2.1 Circular Center Leg (UR- and ER-core)

In case of UR- and ER-cores with circular center leg, the windings are curved along the whole circumference as depicted in Figs. 2e&f. Here, the partial leakage lengths can be derived from the IW and OW leakage length $l_{\sigma,\text{in}}$ and $l_{\sigma,\text{out}}$ with the equations (3) and (4).

$$l_{p,\text{in, circ}} = s_{\text{core}} \cdot \frac{\alpha (\text{rad})}{2\pi} \cdot l_{\sigma,\text{in}}$$
$$l_{p,\text{out, circ}} = \frac{2\pi - s_{\text{core}} \cdot \alpha (\text{rad})}{2\pi} \cdot l_{\sigma,\text{out}} \tag{4}$$

The factor $s_{\text{core}}$ (5) takes the proportion of IW and OW cross section of the core type into account.

$$s_{\text{core}} = 1 \ldots \text{ for UR- and U-core; } \quad s_{\text{core}} = 2 \ldots \text{ for ER- and E-core} \tag{5}$$

The magnetic energy distributions of IW and OW cross section are typically different. Hence, $L'_{\text{g, in}} \neq L'_{\text{g, out}}$ and $l_{\sigma,\text{in}} \neq l_{\sigma,\text{out}}$. Between the IW and the OW cross section, there is a continuous transition region as indicated in Fig. 2f. However, the calculation is based only on two 2D cross sections (IW and OW) for simplicity reasons. Therefore, an abrupt transition from IW to OW cross section has to be chosen. In this paper, the IW-OW transition is assumed to take place at the angle $\alpha = 2 \arcsin \left( \frac{d_1}{b_{\text{leg}} + 2b_{a,1} + 2d + 2a_2} \right)$.

Fig. 2: a&b) Basic 2D cross sections of Double-2D approach. c&d) leakage lengths resulting from the rotated cross sections, e)–h) IW and OW partial leakage lengths of UR-, ER, U-, and E-core.
### 2.2.2 Rectangular Center Leg (U- and E-core)

For U- and E-cores with rectangular center legs as shown in Figs. 2g&h, the partial leakage lengths consist of straight and curved parts. The straight parts of the partial leakage lengths can simply be derived from width and depth of the rectangular center leg. For the curved parts around the center leg edges, the OW-leakage length \( l_{\text{out,rect}} \) with \( d_{\text{leg}} \rightarrow 0 \) is used.

\[
\begin{align*}
    l_{\text{p,in,rect}} &= s_{\text{core}} \cdot d_{\text{leg}} \quad (6) \\
    l_{\text{p,out,rect}} &= (2 - s_{\text{core}}) d_{\text{leg}} + 2 h_{\text{leg}} + l_{\text{out,rect}} \quad (7)
\end{align*}
\]

### 3 Modelling the Leakage Length

For calculating the partial leakage lengths, the curved winding sections are most crucial for accuracy. In the curved winding sections, a more sophisticated model is required than for non-curved winding sections. Hence, this paper focuses on the calculation of the inside-window (IW) and outside-window (OW) leakage lengths resulting from the rotated IW and OW cross sections.

#### 3.1 Fundamental Concept

As mentioned in the introduction, the leakage length is derived from the magnetic energy distribution as leakage inductance depends on the stored magnetic energy. In curved winding sections, the curvature of the magnetic field needs to be taken into account. Hence, an energy weighted mean length is required to correctly scale the leakage inductance per unit length. In this paper, the term "leakage length \( l_\sigma \)" is introduced for this energy weighted mean length. This leakage length refers to the rotation of the transformer cross section and is therefore derived from an axisymmetric arrangement with circular center leg as shown in Figs. 2a–d. Here, the windings are curved over the whole rotational axis. The contribution of the magnetic energy density \( w_{\text{mag}} \) to the total magnetic energy \( W_{\text{mag}} \) increases with the distance to the rotational axis (i.e. increasing values of \( x \)). Since inductance and energy are proportional, the leakage length \( l_\sigma \) can be calculated according to (8) for the considered axisymmetric arrangement.

\[
\begin{align*}
    l_\sigma &= \frac{W_{\text{mag}}}{W'_{\text{mag}}} \frac{1}{\text{symmetry}} \int_{\mathcal{A}} x \cdot w_{\text{mag}}(x, y) \, dA \\
    &\quad \int_{\mathcal{A}} w_{\text{mag}}(x, y) \, dA
\end{align*}
\]

where \( W'_{\text{mag}} \) is the stored magnetic energy per unit length.

#### 3.2 Leakage Length for Axial Field (1D-Approximation)

In case of purely axial leakage flux inside the transformer window as shown in Fig. 1c, the magnetic field \( \vec{H} \) only has a \( y \)-component \( H_y(x) \) depending on the \( x \)-coordinate. Hence, this case is often referred to as 1D-field. In this case, the leakage length (8) simplifies to the known formula (9) mentioned in the introduction. The formula can be found in [18].

\[
\begin{align*}
    l_{\sigma,\text{axial}} &= \pi \cdot \left( d_{\text{leg}} + 2 d_{\text{1,i}} + a_1 + d + a_2 - \frac{a_2 - a_1}{2} \frac{a_1 + a_2 + 4d}{a_1 + a_2 + 3d} \right) \\
    &\quad (9)
\end{align*}
\]

A derivation of the total rotated magnetic energy can be found in [19]. Dividing the total rotated magnetic energy by the energy per unit length according to (8) results in (9). However, the leakage field is only purely axial when 1.) the windings are modelled with a rectangular block and 2.) reach up to the transformer yokes as displayed in Fig. 1c. The former is a common assumption in leakage inductance models. The latter is technically not feasible due to required isolation distances.

#### 3.3 Leakage Length of Fringing Field (2D-Field)

In real transformers, an isolation distance between windings and yokes is required. Fig. 1d shows the magnetic field of a feasible geometry featuring isolation distances with the only assumption of a rectangular winding block. Here, the field contains axial and radial components (fringing field) and the leakage length cannot be calculated with (9).

The presence of a fringing field does not lead to big deviations from the axial flux formula (9) a priori. In fact, with aligned windings of similar dimensions, the deviation is negligible. Significant deviations from the axial flux assumption arise if windings are not aligned and/or have different dimensions.
Fig. 3: Error of the axial flux approximation (9) for the conducted 2D FEM simulations: a) outside-window (OW), b) inside-window (IW). See Tab. II and III for simulation parameters. c) Effect of flux fringing on the leakage radius (11) and leakage length explained based on an exemplary scenario: The effective leakage radius $r_{\sigma,\text{eff}}$ is pushed towards the smaller winding due to the distribution of the stored magnetic energy. The shift in magnetic energy is especially pronounced at the edges of the smaller winding (see inset). A considerable amount of magnetic energy is stored in this area.

Especially the relative height difference of the windings $\Delta h_{\text{rel}}$ (10) leads to a deviation from the axial flux approximation.

$$\Delta h_{\text{rel}} = \frac{\Delta h_{\text{avg}}}{2 \text{ windings}} = \frac{h_1 - h_2}{\frac{1}{2}(h_1 + h_2)} \quad (10)$$

The relative winding height difference $\Delta h_{\text{rel}}$ leads to a deviation between axial flux approximation (9) and effective leakage length (8) as shown in Figs. 3a&b. These figures show the error of the axial flux approximation for the conducted FEM simulations of outside-window (OW) and inside-window (IW) cross section. The simulation parameters are listed in Tabs. II and III in the Appendix.

From a physical point of view, the fringing flux (Fig. 1d) causes the deviation between effective leakage length and the leakage length assuming an axial field. Flux fringing leads to an altered magnetic energy density distribution compared to a purely axial flux.

The deviation between effective leakage length and leakage length assuming axial flux is best explained introducing the leakage radius $r_\sigma$. As the leakage length $l_\sigma$ is effectively a perimeter due to the rotation of the arrangement, a corresponding leakage radius $r_\sigma$ can be defined according to (11).

$$r_\sigma = \frac{l_\sigma}{2\pi} \quad (11)$$

The leakage radius represents the horizontal center of the magnetic energy. The flux fringing effect usually pushes the effective leakage radius $r_{\sigma,\text{eff}}$ towards the smaller winding compared to the leakage radius assuming axial flux $r_{\sigma,\text{axial}}$ as shown in Fig. 3c. Here, the effective leakage radius $r_{\sigma,\text{eff}}$ deduced from (8) is closer to the smaller winding compared to the leakage radius resulting from an axial field $r_{\sigma,\text{axial}}$. Consequently, the resulting effective leakage length $l_\sigma$ is higher in the case of a geometry such as the displayed geometry in Fig. 3c.

This deviation between the effective leakage length and the leakage length resulting from the axial flux approximation (9) is barely discussed in literature. Therefore, two accurate and computationally efficient leakage length models are proposed in the following section.

4 Proposed Leakage Length Models

4.1Empirically Corrected Axial Flux (ECAF) Model

The ECAF model is based on a compact empirical correction to the axial leakage flux approximation (9) and thus very simple to apply. Particularly, the difference $\Delta r$ between effective leakage radius $r_{\text{eff}}$ and 1D-leakage radius $r_{\sigma,\text{axial}}$ shown in Fig. 4a is empirically modelled. This difference is added to the 1D-approximation to obtain the effective leakage length resulting in (12).
Fig. 4: a) Difference between effective leakage radius and 1D-leakage radius \( \Delta r = r_{\sigma,\text{eff}} - r_{\sigma,\text{axial}} \). b) Concept of ECAF model. c&d) Concept of cut line model: A set of representative cut lines \( U = \{C_1, C_2, \ldots, C_K\} \) is defined as integral domain of (16) for IW and OW cross section.

\[
l_{\sigma,\text{ECAF}} = l_{\sigma,\text{axial}} + 2\pi \Delta r_{\text{ECAF}}
\]  

(12)

The concept of the ECAF model is visualised in Fig. 4b. The correction radius according to the ECAF model \( \Delta r_{\text{ECAF}} \) is calculated according to (13)

\[
\Delta r_{\text{ECAF}} = m \left( h_1 - h_2 \right) \left( \sqrt{\frac{h_1 - h_2}{d + 2\pi \sigma_2}} + \sqrt{\frac{h_1 - h_2}{h_1 + h_2}} \right) \approx \Delta r
\]  

(13)

The geometry parameters are defined in Fig. 1a and \( m \) is a prefactor that depends on whether the equation is applied to inside-window (IW) or outside-window (OW) cross section. Eqs. (14) and (15) show the proper prefactors for both cross sections.

Inside-Window (IW): \( m = \frac{1}{42} \)  

Outside-Window (OW): \( m = \frac{1}{26} \)  

(14) and (15)

The correction radius \( \Delta r_{\text{ECAF}} \) is deduced from the performed FEM simulations listed in Tabs. II and III. Here, a wide range of technically sound transformer geometries is covered. These simulations show that the difference between the effective leakage radius and the 1D-leakage radius \( \Delta r = r_{\sigma,\text{eff}} - r_{\sigma,\text{axial}} \) correlates almost linearly with the height difference of the windings \( h_1 - h_2 \) if the rest of the geometrical parameters are constant. Hence, the other geometrical parameters determine the slope of the almost linear correlation between \( \Delta r \) and \( h_1 - h_2 \). The prefactor \( m \) and the terms in brackets in (13) are empirically determined multipliers that match \( \Delta r_{\text{ECAF}} \) to \( \Delta r \).

With the chosen wide range of winding/core parameters, the presence of the transformer yokes and legs do not significantly influence the leakage length. Therefore, these winding-core distances are neglected in the ECAF model. This approach is chosen to keep the model compact and to keep the focus on the major geometric influence, i.e. the height difference of the windings.

4.2 Cut Line (CL) Model

The cut line (CL) model pursues a semi-analytical approach with the target of achieving high accuracy without calculating the complete field distribution. To achieve this, a set of horizontal cut lines \( U \) is defined, along which the field is integrated. This avoids integrating the field over the total 2D cross section. The concept is visualised in Figs. 4c&d. With this approach, \( l_{\sigma} \) (8) is approximated by (16).

\[
l_{\sigma,\text{Cut Line}} = 2\pi \sum_{n=1}^{K} \frac{\int_{C_n} x \cdot w_{\text{mag}}(x, y_n) \, dx}{\int_{C_n} w_{\text{mag}}(x, y_n) \, dx}
\]  

(16)

\( U = \{C_1, C_2, \ldots, C_K\} \) is the set of cut lines and \( K \) is the total amount of cut lines. This set of cut lines \( U \) needs to be chosen such that the leakage length is well approximated.

To achieve high accuracy independent of the geometry, different sets of cut lines \( U \) are defined depending on the considered geometry. Figs. 5a&b illustrate the choice of the cut line set depending on geometric characteristic numbers. The cut line sets are specified below (see Fig. 1a for definition of geometry parameters):

- **Center Line Model**: The center line model uses only one cut line at center winding height

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Fig. 5: Model selection flowcharts of cut line model depending on geometric characteristic numbers. a) inside-window (IW), b) outside-window (OW)

\[ y_1 = h_{yoke} + d_y + \frac{h_1}{2} \]  
\[ y_2 = h_{yoke} + d_y + \frac{h_1}{2} \]  
\[ \text{2-Line Model:} \text{ The 2-line model uses two cut lines at} \]
\[ y_1 = h_{yoke} + d_y + \frac{h_1}{2}, \quad y_2 = h_{yoke} + d_y + h_b \]  

\[ y_1 = h_{yoke} + d_y + \frac{h_1}{2}, \quad y_2 = h_{yoke} + d_y + \frac{3}{8} h_1, \quad y_3 = h_{yoke} + d_y + h_b \]  
\[ \text{3-Line Model:} \text{ The 3-line model uses three cut lines at} \]

\[ y_1 = h_{yoke} + d_y + \frac{h_1}{2}, \quad y_2 = h_{yoke} + d_y + \frac{3}{8} h_1, \quad y_3 = h_{yoke} + d_y + h_b, \quad y_4 = h_{yoke} + d_y \]  
\[ \text{4-Line Model:} \text{ The 4-line model uses four cut lines at} \]

\[ l_{C,in} = w_w \]  
\[ l_{C,in} = w_{12} \]  
\[ l_{C,\text{out}} = d_x + q_{\text{out}} \cdot w_{12} \quad \text{with} \quad q_{\text{out}} = 3 + 2 \frac{h_1 - h_2}{d + (a_1 + a_2)/3} \quad \text{and} \quad w_{12} = d + a_1 + a_2 \]  

Fig. 6: Error of the axial flux approximation, the ECAF model, and the cut line model for the simulated scenarios from Tabs. II and III. The fitted curves are given because some data points are obscured by others. Fits are 3rd order polynomials and determined with the least squares method.
Table I: Performance indicators of the proposed models. The normalised Root Mean Square Error (nRMSE) gives the approximate scale of error to expect from a model.

<table>
<thead>
<tr>
<th>Cross section</th>
<th>nRMSE</th>
<th>Axial Flux (9)</th>
<th>ECAF (12)</th>
<th>Cut Line (16)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IW (Fig. 2a)</td>
<td></td>
<td>5.89%</td>
<td>1.36%</td>
<td>0.65%</td>
</tr>
<tr>
<td></td>
<td>Worst Case Error</td>
<td>17.54%</td>
<td>7.11%</td>
<td>2.14%</td>
</tr>
<tr>
<td>OW (Fig. 2b)</td>
<td></td>
<td>8.15%</td>
<td>0.90%</td>
<td>1.17%</td>
</tr>
<tr>
<td></td>
<td>Worst Case Error</td>
<td>21.84%</td>
<td>2.19%</td>
<td>3.10%</td>
</tr>
</tbody>
</table>

5 Model Validation

Both, ECAF and CL model give a good approximation of the effective leakage length. This can be seen in Figs. 6a & b which show the error of each model as a function of the relative winding height difference $\Delta h_{rel}$ (10) for the performed simulations listed in Tab. II and III. The figure shows that both, ECAF (12) and cut line model (16) result in a significantly lower error compared to the axial flux approximation (9). Tab. I shows the overall performance of the proposed models by listing the normalised root mean square error (nRMSE) and the worst case error. The nRMSE indicates the approximate scale of error to expect from a model and can be calculated according to (23).

\[
nRMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (l_{\sigma, \text{model},i} - l_{\sigma, \text{FEM},i})^2} \frac{\max(l_{\sigma, \text{FEM}}) - \min(l_{\sigma, \text{FEM}})}{\max(l_{\sigma, \text{FEM}}) - \min(l_{\sigma, \text{FEM}})} \quad (23)
\]

Regarding the IW cross section, the cut line model performs especially well with an nRMSE as low as 0.65%. This is because the magnetic energy is confined to the transformer window and therefore, the cut lines give a very good approximation of the total energy distribution. The ECAF model is also relatively accurate within a very big range, which is indicated by the low nRMSE of 1.36%. Only for individual scenarios with very high $\Delta h_{rel}$, the error reaches up to 7.11% for the set of simulated scenarios. Regarding the OW cross section, the ECAF model performs especially well with an nRMSE of only 0.90%. This is because the winding-yoke distances $d_{y,b}$ and $d_{y,t}$ and the outer winding-leg distance $d_{x,o}$ as defined in Fig. 1a do not exist in the OW cross section. In the ECAF model, these distances practically represent disturbances and therefore the model is very accurate when these parameters are absent. The cut line model also performs quite well with an nRMSE of 1.17%. The accuracy is slightly worse for the OW cross section because the magnetic energy is not confined to the transformer window in the OW cross section.

The previously assessed performance of the models is reflected in detail by the histograms presented in Fig. 7. The histograms show the error density distribution of the simulated scenarios.

![Fig. 7: Error range histogram for IW cross section. a) 1D-Approximation, b) ECAF model, c) CL model](image-url)
6 Conclusion

The leakage length which is required for analytical leakage inductance modelling of curved winding sections has been investigated. More than 6000 performed FEM simulations showed that a height difference of the windings leads to error if the leakage length is calculated with the known formula that assumes purely axial leakage flux, i.e., a 1D-field. The error is caused by the altered magnetic energy density distribution of a fringing field, i.e., 2D-field.

The proposed ECAF model is based on a compact empirical modification of the axial flux approximation. With this approach, the error is significantly decreased manifesting in an nRMSE as low as 1.36 % and 0.90 % for the simulated inside-window (IW) and outside-window (OW) scenarios, respectively. The ECAF model is compact, easy to implement, and rapidly computable. Considering that the outside-window winding section typically exceeds the inside-window winding section, this model becomes even more relevant.

The proposed cut line model is even more accurate for the inside-window scenario. The cut line model is a semi-analytical model yielding an nRMSE of 0.65 % for the IW scenarios and 1.17 % for the OW scenarios. The increased accuracy comes at the cost of higher complexity and higher computational effort.

Both proposed models can be implemented in an optimisation procedure. The model choice depends on the desired trade-off between accuracy and computational effort. Using the ECAF model for IW and OW cross section will lead to high accuracy and very low computational effort. If accuracy is the most important criterion, the cut line model should be used for the inside-window cross section and the ECAF model for the outside-window cross section.

Appendix

More than 6000 FEM simulations are performed to derive the proposed models. Both, inside-window (IW) and outside-window (OW) cross sections are evaluated. The geometrical parameters in Tabs. II and III are chosen such that a wide variety of technically sound geometrical winding/core parameters are covered. Only extreme aspect ratios are avoided.

The OW cross section is approximated with a center core leg of infinite height as FEM simulations have shown that the difference in leakage length resulting from a center leg of finite height and a center leg of infinite height is negligibly small.

Table II: Geometrical parameter range of inside-window (IW) and outside-window (OW) simulations. A wide range of technically sound transformers is covered, only extreme aspect ratios are avoided. See Fig. 1a for definition of geometry parameters.

<table>
<thead>
<tr>
<th>Cross section</th>
<th>( \Delta h_{rel} ) (10)</th>
<th>( \frac{b_1-b_2}{\sqrt{d+(a_1+a_2)/2}} )</th>
<th>( \frac{d}{(w_1+a_2)/3} )</th>
<th>( \max(a_1, a_2) )</th>
<th>( \min(a_1, a_2) )</th>
<th>( \frac{(b_1+b_2)/2}{d+(a_1+a_2)/2} )</th>
<th>Simulated scenarios</th>
</tr>
</thead>
<tbody>
<tr>
<td>IW (Fig. 2a)</td>
<td>0–0.5</td>
<td>0–4</td>
<td>0.6–5</td>
<td>1–5</td>
<td>1.06–20</td>
<td>3030</td>
<td></td>
</tr>
<tr>
<td>OW (Fig. 2b)</td>
<td>0–0.5</td>
<td>0–4</td>
<td>0.6–5</td>
<td>1–5</td>
<td>1.06–20</td>
<td>3030</td>
<td></td>
</tr>
</tbody>
</table>
Table III: Winding/core parameter range of IW and OW simulations. Parameters have been set randomly within the given range. See Fig. 1a for definition of geometry parameters.

<table>
<thead>
<tr>
<th>Cross section</th>
<th>(d_{x1}/(a_1+b_2)/3)</th>
<th>(d_{y1}/(a_1+b_2)/3)</th>
<th>(d_{y2}−d_{y1}/(a_1+b_2)/3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IW (Fig. 2a)</td>
<td>0–0.2</td>
<td>0.2–3.6</td>
<td>0.01–0.44</td>
</tr>
<tr>
<td>OW (Fig. 2b)</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

References
