Sensor Design for a Current Measurement System with High Bandwidth and High Accuracy Based on the Faraday Effect

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Keywords
≪Current Sensor≫, ≪Sensor≫, ≪Transducer≫, ≪Frequency-Domain Analysis≫

Abstract
This paper presents the design of the optical system of a current sensor with a wide bandwidth and a high accuracy. The principle is based on the Faraday effect, which describes the effect of magnetic fields on linearly polarized light in magneto-optical material. To identify suitable materials for the optical system the main requirements and specification are determined. A theoretical description of the optical system shows a maximal applicable magnetic field frequency due to the finite velocity of light inside the material. Hence, the theoretical characterisation of the system implies practical boundaries on the optical material and its specifications. Finally, two different magneto-optical crystals, Terbium Gallium Garnet and Cadmium Manganese Telluride, are investigated, assembled and put into an optical system prototype.

1 Introduction
Highly accurate current measurement systems with high bandwidths are for example required for precise operation and control of power switches in injection and extraction kicker magnet systems. These systems, as for example used in CERNs particle accelerators, require the measurement of fast rising currents with more than 150 kA/μs [1] to generate the required pulsed magnetic fields. The actual current rise times in such applications from 0.1 μs up to 1 μs [2, 3] lead to bandwidth requirements between some kHz and several MHz. Furthermore, high measurement accuracy in the range of several ppm is necessary to reliably control and operate these systems.

Classical current measurement concepts, such as shunts, current transformers, Rogowski coils and Hall effect based systems are usually capable of measuring bandwidths from DC or a couple of Hz up to several tens of MHz [4, 5, 6, 7, 8, 9]. The uncertainty of these systems is usually in the range of 0.1 % up to 5 % [6]. However, the accuracy of these systems decreases usually significantly at high frequencies. For instance, the uncertainty of a shunt at low frequencies can be in the ppm range but increases for higher frequencies up to several percent [9].

Fig. 1: The current measurement system is shown as the concatenation of transfer functions of multiple subsystems. The optical system is built using an intensity modulation scheme with two polarising stages and a magneto-optical crystal (sensor) in between. A single mode laser diode (λ = 660 nm) is used as light source.
Table I: Current sensor requirements

<table>
<thead>
<tr>
<th>Specification</th>
<th>100 A ... 10kA</th>
<th>DC... ≥ 10MHz</th>
<th>&lt; 0.1%</th>
<th>&lt; 25 ppm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current range</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Frequency bandwidth</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measurable pulse rise time (5% - 95%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full bandwidth uncertainty (accuracy)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reproducibility error (precision)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A promising approach for measuring a current with high accuracy and high bandwidth is based on the Faraday effect. The Faraday effect describes the rotation of the plane of polarisation (PoP) of linearly polarized light inside a magneto-optical material. The rotation of the PoP is proportional to an externally applied magnetic field in the direction of the optical path. This magnetic field is generated by the current to be measured. Hence, a quasilinear relation can be expected between the rotation of the PoP and the current. In addition, Faraday effect based current measurement systems are insensitive to electromagnetic interference (EMI) and they provide good isolation from the power rail.

Over the years, different current measurement concepts utilising the Faraday effect have been investigated targeting diverse specification and using different material, geometrical shapes and different signal processing schemes [10, 11, 12, 13, 14]. In contrast to the system approach of this paper, most of the investigated systems limit their scope to either high current amplitudes, highly accurate measurements or high bandwidths. Furthermore, the rotation of the PoP has usually been limited to 180°, such that the initial current and magnetic field strength are explicitly reproducible. To achieve the requirements given in Tab. I, i.e. aiming at high accuracy and wide bandwidth, a concept using rotation angles larger than 180°, as presented in [15], is investigated in this paper. In [15], the sensor is designed using a bismuth substituted RIG-film (ferrimagnetic) with a saturation rotation of around 150° mm⁻¹. The authors use a total Faraday rotation of 3300° resulting in an comparably high accuracy of 4.9 ppm. The focus in [15] lies on the limitations caused by the analog signal processing. In contrast, this paper focuses on the characterization and modelling of the optical system to derive its transfer function (cf. Fig. 1). Eventually, a complete analytical model of the current measurement system is pursued. Moreover, different magneto-optical materials are presented in this paper.

In section 2 the optical system is explained including the basic concept of the measuring process, the identification of possible error sources and the choice of the magneto-optical material. In section 3 an approach chosen to model the optical system and derive the transfer function is introduced and applied to the actual chosen material. Eventually, the paper concludes in section 4.

2 Optical System

The optical system, as schematically depicted in Fig. 1, denotes the physical measuring part of the current measurement system. It contains a laser diode emitting linearly polarised light, two (or more) polarisers for the intensity modulation and the magneto-optical material. The interface between the optical system and the subsequent optical-to-electrical conversion system is a photodiode. The optical system maps the physical measurand (current, magnetic field strength) to its optical and further to its electrical counterpart (intensity modulated polarised light, photodiode current).

2.1 Faraday Effect

The effect allowing to map the magnetic field strength to the polarisation state of a light beam is the Faraday effect. The rotation angle of the PoP \( \Theta \) (1) and the Verdet constant \( \Psi(\lambda) \) (2) are described by this effect with

\[
\Theta = \mathcal{V}(\lambda) \cdot \mu_0 \cdot \mu_r \cdot \int_0^L H \, dl \\
\mathcal{V}(\lambda) = -\frac{e}{m_e} \cdot \frac{\lambda}{2c} \cdot \frac{dn_r}{d\lambda}
\]

where \( H \) is the magnetic field strength parallel to the optical path and \( L \) the optical path length exposed to the magnetic field. The Verdet constant characterises the material’s optical activity for an incident wave with wavelength \( \lambda \). The wavelength dependency of the Verdet constant \( \Psi(\lambda) \) is described in (2) [16], where \( e \) is the charge of an electron, \( m_e \) the mass of an electron, \( c \) the speed of light in vacuum and \( n_r \) the materials refractive index.
2.2 Measurement Concept

As depicted in Fig. 1, the rotation angle of the PoP is analysed by modulating the intensity of the light beam. After the PoP of a beam of linearly polarized light is rotated inside the magneto-optical material, a polariser (analyzer) modulates it to represent the intensity as a sinusoidal signal. Applying Malus’s law, the intensity $E_{e1}$ at the output of the analyzer is described as

$$E_{e1} = \alpha \cdot E_{e0} \cos^2(\Theta_i)$$

(3)

where $E_{e0}$ is the intensity of the beam before the initial polariser, $\alpha$ a factor for the transmission losses and $\Theta_i$ the angle between the analyzer’s axis of polarisation and the angular direction of the PoP. Figure 2 represents the output of the optical system after using for example a polarising beamsplitter with two channels with an offset of 45°.

2.2.1 Multiple Rotation Concept ($\Theta > 180°$)

The measurement resolution can be improved by relatively increasing the rotation angle of the PoP to the current to be measured. From this point of view, it is beneficial if the rotation angle is as large as possible which poses a critical restriction to the chosen magneto-optical material (cf subsection 2.3). This includes specifically that the rotation of the PoP exceeds 180°. As shown in Fig. 2, after the rotation angle of the PoP exceeds 180° the output at the analyzer becomes ambiguous even if multiple channels are observed. Hence, a counting mechanism needs to be introduced to distinguish between the different rotation intervals.

Considering (1) and (2), the rotation of the PoP can be affected in multiple ways:

- Choice of magneto-optical material: The choice of the material determines the Verdet constant and hence the primary influence on the rotation.
- Choice of center wavelength: The Verdet constant of a certain material is wavelength dependent. Therefore, decreasing or increasing the wavelength results in more or less rotation per unit magnetic field strength, respectively.
- Length of the optical path: The length of the optical path through the magneto-optical material increases the final rotation due to the relation stated in (1). On the other hand, optical transmission through longer optical pathes damps the intensity due to absorption and scattering.
- Magnetic field strength: The maximal magnetic field strength determines the maximum angle of rotation. By improving, for example, the conductor geometry the maximal magnetic field strength could be increased.

A further advantage of having a rotation angle larger than 180° is the depression of the relative measurement error due to imprecision in the determination of the rotation. For simplifying the analysis one needs to consider the measurement of the rotation angle and the mechanism to detect the crossing of 180° to be two independent events. Hence, measuring the angle of the part of the rotation which exceeds multiples of 180° is a single event bound to a certain accuracy. This event and its accuracy do not differ from the measurement of a PoP which does not exceeded 180°, meaning that the measurement error only applies to the last interval of 180°. Therefore, the ratio between the absolute measurement error and the absolute angle of rotation decreases if the total rotation is increased. Assuming a percentaged measurement error $e_m$, the ratio between the absolute error $e_t$ and the total rotation $\Theta_i$ can be expressed as

$$\frac{e_t}{\Theta_i} = \frac{e_m \cdot (\Theta_i \mod 180°)}{\Theta_i}$$

(4)
The ration between the absolute error and the total rotation is divided in half for every additional 180° of rotation. If the measurement error is static (e.g. polariser misalignment) by representing an offset of ΔΘ, the additional rotation is decreasing this ratio even further resulting in $e_i/\Theta = ΔΘ/\Theta_i$. Since more rotation leads to a better resolution of the measurement, the error per magnetic field measured decreases and the overall system performance improves.

A potential restriction to more optical rotation is the urge for a faster analog-to-digital (ADC), comparator circuit and transimpedance circuitry. Since the current to measure ramps at with the same $\frac{di}{dt}$ but the angular velocity of the optical rotation increases, the time interval in which at least one measurement has to be captured shortens. Hence, designing an accurate and fast signal processing stage is crucial to benefit from the concept of multiple turns.

### 2.2.2 Optical System Error Sources

Deviations from the ideal measurement response can occur due to tolerances and imperfections in the above mentioned system parameters which influence the rotation of the PoP. For the parameter dependencies, as shown in Fig. 3, and the optical system setup three error classes have been identified.

1. **Polariser misalignment:** If the polariser and analyser are misaligned by a static angle $β$, the reference for the intensity modulation is wrong. Hence, the interpretation of the signal is set of against the expected result. Observing the analysers output with a total rotation $Θ_i$, this error adapts (3) to

$$E_{e1}(t) = α \cdot E_{e0} \cos^2(Ω_i(t) + β)$$

(5)

2. **Rotation angle error:** Manufacturing tolerances and environmental drifts in the magneto-optical material and in the light pulse characteristics affect the total rotation angle of the PoP. Examples for this effect are tolerances in the length of the crystal, drifts in the light’s wavelength due to temperature changes, and additional rotation in the light guiding material (fibre optical cables). Since the total rotation depends on the Verdet constant, the length of the magneto-optical material and the magnetic field strength, the dependency of the error can be written as

$$\Theta(t) = Θ_i(t) + Θ_Δ(t) = B(t) \cdot e_{opt} \cdot (V' + V'_err) \cdot (L + L_{err})$$

$$\Theta_Δ(t) = B(t) \cdot e_{opt} \cdot (V'L_{err} + V'_{err}L + V'_{err}L_{err})$$

(6)

(7)

where the subscript $err$ denotes the error term of the Verdet constant and the length and $e_{opt}$ is the unit vector in direction of the optical path. Hence, the time-dependent output characteristic at the analyser from (3) changes to

$$E_{e1}(t) = α \cdot E_{e0} \cos^2 \left( (Ω_i + Ω_Δ)(t) \right)$$

(8)
Note, that this error source exhibits the same effect as the polariser misalignment for the DC case with a constant magnetic field.

3. **Transmission rate error:** Due to tolerances and drifts in the length of the magneto-optical material and the light’s wavelength, the transmission through the optical crystal is also affected. Those tolerances affect the amplitude of the intensity with a scaling factor $\alpha$ introduced in (3). The transmission rate of the crystal is wavelength dependent and longer crystals have higher losses due to scattering and absorption. The wavelength dependent parameter $\alpha$ is described by the Beer-Lambert law according to

$$\alpha = a(\lambda) \cdot \exp (-b(\lambda) \cdot d_L)$$

where $a$ and $b$ depend on the material and $d_L$ is the length of the material passed by the light. Assuming tolerances in the material length and the wavelength the output characteristic at the analyser is

$$E_{\text{el}} = a(\lambda + \Delta\lambda) \cdot \exp \left[ -b(\lambda + \Delta\lambda) \cdot (d_L + \Delta d_L) \right] \cdot E_{\text{el0}} \cdot \exp \left( \Theta(t) \right)$$

Summarizing the three error sources, the time-dependent output at the analyser can be described as

$$E_{\text{el}} = a(\lambda + \Delta\lambda) \cdot \exp \left[ -b(\lambda + \Delta\lambda) \cdot (d_L + \Delta d_L) \right] \cdot E_{\text{el0}}$$

$$\cdot \exp \left( \Theta(t) \right) \cdot \left[ B(t) + \varepsilon_{\text{opt}} \cdot (\varepsilon_{\text{err}} + \varepsilon_{\text{errL}} + \varepsilon_{\text{errLerr}}) + \beta \right]$$

Reducing the impact of these error sources is addressed in different parts of the complete current measurement system design. The polariser misalignment error source is a matter of calibration. Hence, when setting up the system, the actual output of the analyser needs to be adjusted to fit with the expected output. Tolerances issuing rotational errors must be avoided by using accurate measures to define the respective materials and a stable light source. Likewise, drifts in runtime parameters (e.g. temperature drifts which lead to wavelength shifts) need to be addressed either by implementing suitable control loops (e.g. TEC drivers, laserdiode power monitoring) or sufficient stabilisation of the parameters (e.g. passive cooling).

A concept to make the output independent of the input intensity $E_{\text{el0}}$ and reduce the impact of potential drifts in the transmission has to be applied. Since the intensity modulation relies entirely on the input intensity getting rid of this dependency has to be pursued in the data processing stage. For this project, the sum over difference principle, as used in [15], is applied.

### 2.3 Magneto-optical Material

#### 2.3.1 Material Evaluation Process

The magneto-optical material is a key component of the optical system. It defines the performance of the measurement system and introduces limitations and requirements to the preceding and subsequent stages. In general, the material used for the sensing element needs to comply with the following list of requirements.

1. Large Verdet constant: A large Verdet constant sets the basis for a large rotation of the PoP.
2. High optical transmission: The longer the optical path, the less transmission can be expected at the output. On the other hand, a long optical path results in a larger rotation of the PoP. A trade-off between optical path length and suitable transmission rate needs to be found.
3. Bandwidth: The complete setup needs to provide a large bandwidth to support low (Hz range) and high (MHz range) magnetic field frequencies.
4. Linearity: For the reproduction of the initial magnetic field, the material needs to behave linearly in the measured range. For the reproducibility of the measurement saturation and hysteresis effects need to be minimised.

Those requirements pose optimisation goals for the design of the optical system. The optical rotation, the total transmission and the measurement bandwidth are the design specifications of the optical systems performance. The relation between those specifications is depicted in Fig. 3. In general, a large Verdet constant is the primary requirement since it directly increases the rotation of the PoP. On the other hand, the Verdet constant is a material dependent value and can only be adapted in a very limited range by adjusting the center wavelength of the light source. Contrariwise, increasing the light sources wavelength...
will decrease the total transmission. Obviously, a longer optical path decreases the transmission since there is more material in which the beam can be scattered and absorbed. It is worth to note, that coupling losses (Fresnel losses) account for a substantial portion of the complete transmission loss (> 4% per surface) using gradient-index (GRIN) lenses. While it is possible to detect very low intensities with photodiodes, a subsequent transimpedance amplifier would need to convert a very small current drawn from the photodiode into a voltage. In general, higher photodiode currents lead to higher possible bandwidths of the transimpedance amplifier and noise considerations are simplified. The necessary measurement bandwidth is defined by the angular velocity of the rotation of the PoP and further limited by the optical path length as described in section 3.

2.3.2 Magneto-Optical Material Decision

Two paramagnetic crystals, namely terbium gallium garnet (TGG) and cadmium manganese telluride (CdMnTe) are presented in this section. These two magneto-optical materials are characterised along the requirements introduced in the previous section. Both materials are tested in two different lengths in order to prove the presented theory. The relevant specifications are listed in Tab. II. Furthermore, the Verdet constant of examples for ferrimagnetic and diamagnetic materials is given for comparison.

Paramagnetic materials do not show saturation effects even if they are exposed to high magnetic field amplitudes. Due to the inherent saturation effects of diamagnetic and ferrimagnetic materials, the optical current measurement sensor needs to be designed such that $B < B_{\text{sat}}$.

The magnetic field strength inside the optical path is subject to the actual location of the magneto-optical sensor with respect to the current carrying conductor, hence it is a design parameter. On the other hand, this means that a ferrimagnetic and diamagnetic material needs to tradeoff the maximal field with a larger Verdet constant and a long optical path. Examples for ferrimagnetic material are yttrium iron garnet (YIG) or gallium yttrium iron garnet (Ga:YIG) which show saturation effects at 0.178 T and 0.04 T [17], respectively. On the other hand, the Verdet constant ($\Psi_{\text{YIG}} \approx 126 \text{deg}/(\text{T}\cdot\text{mm})$ [18], $\Psi_{\text{Ga:YIG}} \approx 353 \text{deg}/(\text{T}\cdot\text{mm})$ [17]) of these material is comparable with CdMnTe and the transmission for small (< 5 mm) sample lengths is reported to be high (> 95%) in the infrared region between 1100 nm and 1400 nm. Taking the saturation into account, the maximal rotation per length is $\Psi_{\text{YIG}} \approx 21.4 \text{deg}/\text{mm}$ and $\Psi_{\text{Ga:YIG, max}} \approx 14.5 \text{deg}/\text{mm}$ which makes large sensor designs necessary to achieve the same performance as with the paramagnetic materials. Diamagnetic material shows significantly lower Verdet constants than TGG or CdMnTe. Examples for those materials are yttrium aluminium garnet (YAG) ($\Psi_{\text{YAG}} = 0.336 \text{deg}/(\text{T}\cdot\text{mm})$), BK-7 glass ($\Psi_{\text{BK-7}} = 0.246 \text{deg}/(\text{T}\cdot\text{mm})$) and Dynasil 1001 ($\Psi_{\text{Dynasil1001}} = 0.199 \text{deg}/(\text{T}\cdot\text{mm})$) [18]. Note, that these values are given for a center wavelength of $\lambda = 632.8 \text{nm}$. Designs with these materials would require very long sensors and consequently a high transmission which makes a design, given the requirements from Tab. I, unfeasible.

TGG is a widely used material for optical Faraday rotators. Due to its popularity in other designs and, as a consequence, the broad documentation, it also serves as a reference value for the evaluation of the

**Fig. 4:** (a) Crystal (CdMnTe, TGG). (b) Full assembly. The sensing element is attached to two optical fibers with GRIN lenses. The optical fibers are polarisation maintaining to prevent polarisation shifts due to mechanical stress or other environmental influences. The crystal is glued into the housing to prevent the setup from becoming misaligned.
Fig. 5: During the entry of light ($t_0$) into the magneto-optical material and its exit ($t_1$), the magnetic field $H(t)$ changes by a factor $\Delta H_i$. Hence, the rotation angle represents the integration over the complete period the light remains inside the crystal. By observing the rotation angle for multiple points in time a *moving average* can be identified.

CdMnTe has been intensively investigated in the past decades [19, 20, 21, 22] as well as modification of it such as for example Cadmium Manganese Mercury Telluride (CdMnHgTe) [23]. It is available in polycrystalline and crystalline form [24]. Since the polycrystalline form suffers from significantly higher light-scattering, the pure crystalline form has been chosen for this sensor design. In general, CdMnTe offers a considerably higher Verdet constant compared to TGG. The composition of (Cd$_{1-x}$Mn$_x$Te) can be altered to achieve different transmission and Verdet constant values for specific wavelengths. Since the absorption of light in CdMnTe is very high it is necessary to adapt the wavelength specifically to the chosen composition and to very careful choose the physical dimension of the crystal (optical path). Counteracting the high transmission loss with higher light input power will eventually lead to higher thermal losses in the material itself. Hence, samples with over 20 mm length are hardly suitable and will need special care in terms of external cooling of the magneto-optical material.

### 3 Sensor Transfer Function

As depicted in Fig. 1, the optical system including the sensor is modeled with a transfer function and as a part of a larger system. The final aim is to fully determine the transfer function of all parts of the system which enables to comprehensively optimise the sensor. The main input to the optical system is the magnetic field $H(t) = \frac{1}{\mu_0 \mu_r} \cdot B(t)$. The measurable output of the systems is the intensity of the laser.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Cd$<em>{57}$Mn$</em>{43}$Te</th>
<th>Cd$<em>{57}$Mn$</em>{43}$Te</th>
<th>TGG</th>
<th>TGG</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optical path length</td>
<td>2.2</td>
<td>6.6</td>
<td>2.2</td>
<td>10.0</td>
<td>mm</td>
</tr>
<tr>
<td>Material Verdet constant $V_{\text{mat}}$</td>
<td>150</td>
<td>150</td>
<td>6.6</td>
<td>6.6</td>
<td>deg/(T·mm)</td>
</tr>
<tr>
<td>Absolute Verdet constant $V_{\text{abs}}$</td>
<td>330</td>
<td>990</td>
<td>14.52</td>
<td>66</td>
<td>deg/T</td>
</tr>
<tr>
<td>Absolute transmission rate</td>
<td>54</td>
<td>45</td>
<td>&gt;99</td>
<td>&gt;99</td>
<td>%</td>
</tr>
<tr>
<td>Refractive index</td>
<td>2.9</td>
<td>2.9</td>
<td>1.98</td>
<td>1.98</td>
<td>-</td>
</tr>
<tr>
<td>Magnetic Material Class</td>
<td>Paramagnetic</td>
<td></td>
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</tr>
</tbody>
</table>

Table II: Magneto-optical crystal specification: The optical and wavelength dependent values are given for a incident laser wavelength of 660 nm.
beam modulated by a polariser setup. The modulated intensity indicates the angle of rotation caused by
the magneto-optical material.

In previous publications [12, 25], the frequency dependency of the Faraday rotation to an applied magneto-
tific field has been analysed. In [12], the time-bandwidth product (TBP) for gaussian shaped laser pulses
is used to determine the measurement bandwidth of the sensor. The TBP (12) describes the relation
between the cutoff frequency \( f_{3dB} \) (13) of the optical system and the laser pulse length \( \tau_{light} \)
\[
TBP = \tau_{light} \cdot f_{3dB} \quad (12) \\
f_{3dB} = 0.44 \cdot \frac{1}{\tau_{light}} = 0.44 \cdot \frac{c}{L \cdot n_t} \quad (13)
\]

where \( n_t \) is the refractive index of the material, \( L \) the length of the optical path and \( c \) the speed of light in
vacuum. For a gaussian shaped laser pulse the TBP is \( \approx 0.44 \) [12, 26]. For a long optical path \( L \) inside
magneto-optical materials this approach limits the measurement bandwidth.

A different effect has been investigated in [25], focusing on the frequency dependency of Cd\(_{1-x}\)Mn\(_x\)Te
due to magnetisation effects inside the material. For different compositions a different frequency be-

caviour can be expected, whereas the Faraday rotation increases with an increasing amount of mangan-
ese \((x)\). This cutoff frequency has been reported to be around 2 GHz for the Cd\(_{55}\)Mn\(_{45}\)Te composition

3.1 Analysis of the Filter Model

In the sensor, a time-variant magnetic field \( H(t) \) is integrated over the length of the magneto-optical
crystal, as described in (1). The sensor frequency response can be modeled and analyzed as a finite
impulse response (FIR) filter implementing a moving average, as shown in Fig. 5. Every \( \Theta(t) \) at the
output of the system represents the sum of all fractions of the rotation over the complete transmission
time (history). The model is predicated on the fact that the speed of the light passing the sensor is finite
and needs to be taken into account. The speed \( v_{light} \) at which light travels inside a material with the
refractive index \( n_t \) and the transmission time \( \tau_{light} \) are calculated according to
\[
v_{light} = \frac{c}{n_t} \quad (14) \\
\tau_{light} = \frac{L \cdot n}{c} \quad (15)
\]

The applied magnetic field, and hence the resulting Faraday rotation, change during \( \tau_{light} \) by the value
\( \Delta H \) and \( \Delta \Theta \), respectively. Assuming that one could measure every distinguishable package of protons
independently, a moving average for the total rotation of the PoP results, as depicted in Fig. 5. For each
light sample, the averaging window with size \( \tau_{light} \) moves one sample step \( T \) to the right. For this model
the sample time \( T \) has been set to a fraction \( m \) of \( \tau_{light} \) inside the magneto-optical material approximating
the continuity of light to discrete values. The system and its transfer function can be described by:
\[
h[n] = \frac{1}{m+1} \sum_{i=0}^{m} b_i \cdot \delta[n-i] \quad (16) \\
H(z) = \frac{1}{m+1} \sum_{i=0}^{m} b_i \cdot z^{-i} \quad (17)
\]

with \( b_i = \mathcal{I}^i \cdot L = \text{const} \forall i \). The applied magnetic field as the input parameter \( x[n] \) to the above described
system and the respective transformation can be described as
\[
x[n] = \frac{\mu_0 I}{2\pi r} \sin(\omega \cdot n) \quad (18) \\
X(z) = \frac{\mu_0 I}{2\pi r} \frac{z \sin(\bar{\omega})}{z^2 - 2z \cos(\bar{\omega}) + 1} \quad (19)
\]

with \( \bar{\omega} = \frac{2\pi f}{N} \). Eventually, the convolution of the input signal with the system transfer function describes
the Faraday rotation \( \Theta(z) \) as
\[
\Theta[n] = h[n] \ast x[n] \quad (20) \\
\Theta(z) = H(z)X(z) = \frac{\mu_0 I \mathcal{I} L}{2\pi r} \frac{1}{m+1} \sum_{i=0}^{m} z^{-i} \frac{z \sin(\bar{\omega})}{z^2 - 2z \cos(\bar{\omega}) + 1} \quad (21)
\]

The transfer function eventually describes the sensor material’s time-dependent behaviour analytically.
Applying this relation to actual magneto-optical material allows the identification of suitable material
with respect to its material dependent and geometrical parameters.

3.2 Transfer Function Results

For the identified sensor materials, CdMnTe and TGG, the analysis of the sensor transfer function is
performed. Figure 6 shows the calculated cutoff frequencies for the filter model for different crystals and
their respective lengths. Inserting the length of the TGG crystal from Tab. II to the limiting TBP formula, the cutoff frequency is expected to be at $\approx 32 \text{GHz}$ for the 2.2 mm and $\approx 7.5 \text{GHz}$ for the 10 mm crystal. For the CdMnTe crystals, the cutoff frequency (TBP approach) is expected to be at $\approx 20 \text{GHz}$ for the 2.2 mm and $\approx 8 \text{GHz}$ for the 6.6 mm crystal. The calculated values match with the results received from the TBP approach and the model from [26].

For bulk material with short (mm range) to medium (<100 mm) optical path length the low-pass behaviour of the magneto-optical material poses no crucial limitation. For CdMnTe the cutoff frequency due to magnetisation effects, as explained at the beginning of section 3 and [25], has been added for comparison (red line). For samples longer than 40 mm the transmission delay limits the design’s maximal bandwidth, for shorter samples the magnetisation effect is dominant and prevents higher bandwidths.

Finally, the analysis shows the tradeoff between the crystal length, the applicable bandwidth, the maximal rotation and hence the accuracy. Longer crystals lead to more rotation and hence a better ratio of applied current to received rotation. On the other hand, the maximal possible bandwidth decreases with a longer optical path.

Consequently, both materials are suitable for long sensor designs allowing a larger absolute rotation of the PoP. Considering Fig. 3, a trade-off between the maximal frequency and the optical path length can be disregarded for the current specifications given in Tab. I. The length of the crystal is defined by the required rotation angle of the PoP and the transmission.

4 Conclusion

This paper presents the design of an optical system of a highly accurate and wide bandwidth current measurement system. The performance of the magneto-optical material is characterised by its length, the materials Verdet constant (optical activity) and the total transmission of the applied optical input power. Eventually, the combination of all requirements and the specifications from the material lead to an optimisation process for the material choice.

Given the requirements for the system, two magneto-optical material have been chosen. Both materials are paramagnetic to prevent the occurrence of saturation effects and achieve high linearity in a large range of possible magnetic field amplitudes. Terbium Gallium Garnet (TGG) has been chosen to provide a reference for the optical system and make it comparable to other systems since the material is widely used and documented. It provides a large transmission rate even with very long samples trading-off the rather low Verdet constant. Cadmium Manganese Telluride (CdMnTe) provides a very large Verdet constant but it shows large absorption and scattering effects reducing the actual transmission of the input power.

Moreover, this paper presents a simple model to describe the frequency behaviour of the magneto-optical material. The model presents a further tool to evaluate the suitability of the magneto-optical material for the complete system. The sensing element has been depicted as high order FIR filter implementing a moving average over the light pulse transmission time window. Using this model, TGG shows a cutoff frequency...
frequency of 32 GHz and 7.5 GHz for the 2.2 mm and the 10 mm samples, respectively. In contrast, CdMnTe shows lower possible frequencies with 20 GHz and 8 GHz for the 2.2 mm and the 6.6 mm samples, respectively. For the specification given in this design, the cutoff frequencies pose no further limitation on the magneto-optical material and the analysis shows that a trade-off between the absolute transmission and the optical path length regarding the current measurement bandwidth is not necessary. The length of the magneto-optical material is defined by the necessary rotation angle of the PoP and the transmission rate.

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References