Comparison of Analytical Transformer Leakage Inductance Models: Accuracy vs. Computational Effort

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Abstract
Fast and accurate models of design and operating parameters are crucial for optimisation-based design of power electronic converters. The leakage inductance of transformers is one of these design parameters. This paper compares various analytical 1D and 2D leakage inductance models, and assesses them with respect to a trade-off between accuracy vs. computational effort. The computed leakage inductance per length values are compared to 2D FEM simulation results to determine the model accuracy, whereas the calculation times are extracted as indicator of each model’s computational effort. The considered models are applied to six existing transformer geometries and compared to measurements.

Roth’s model is the most accurate of the considered models, Rogowski’s model is the fastest model, and Margueron’s model is the most versatile as it takes the magnetic permeability of the core into account.

1 Introduction
Leakage inductance is an important parameter of transformers in many isolated power electronic converters and can significantly influence the operation of the system. Requirements on transformer leakage inductance depend on the topology of the converter. In some converter topologies, the leakage inductance should be as small as possible to minimise losses. The flyback converter is an example of these kinds of systems (see e.g. [1]). In other topologies, the leakage inductance of the transformer can be exploited to replace separate inductors. Thus, the power density is increased, and component costs are saved. This concept is frequently applied in resonant converters (see [2]). In dual active bridge converters, the leakage inductance is required to shape the current and to enable zero voltage switching [3].

In the design stage of a converter, the operating and design parameters are usually determined in an optimisation, before the components are physically built. Such optimisations require accurate models for all relevant converter parameters such as currents, voltages, losses, and, in the case of isolated converters, the leakage inductance of the transformer. The models are evaluated several thousand times during the optimisation process (see e.g. [4]). Therefore, the models need to be computationally efficient and deliver accurate results within reasonable times.

The leakage inductance of transformers can be calculated by numerical and analytical calculation methods. Numerical methods such as the finite element method (FEM) and the boundary element method (BEM) are highly versatile and highly accurate. Their disadvantage is that they are very time-consuming, as a vast number of equations needs to be solved. In contrast, analytical models such as those proposed by Dowell [5] and Rogowski [6] are usually restricted to their assumed geometry and subject to simplifications, which may reduce the accuracy of the result. However, analytical models are very fast to compute, which makes them more suitable for optimisations than numerical models.

Various reviews on leakage inductance models have been published. For example Margueron et al. [7] and Lambert et al. [8] provide a good overview on 2D leakage inductance model approaches for
low-frequency applications. Guillod et al. [9] compare various models for transformer equivalent circuits with respect to statistical parameter uncertainties. Doebbelin et al. [10] compare the accuracy of Rogowski’s model and the mean geometric distance (MGD) model for various winding-interleavings. Fouineau et al. [11] compare the accuracy and calculation times of a 1D model (Dowell), and a ”2x2D model” (mathematically based on Margueron’s model) to 3D FEM-simulations. However, a comparison of more than two leakage inductance models applied to a large number of transformers with respect to a trade-off between computational effort and accuracy is still missing.

Hence, this paper provides an extensive comparison of 1D and 2D leakage inductance models and assesses them with respect to a trade-off between accuracy and computational effort. To begin with, an overview and categorisation of existing models and their underlying assumptions, approaches, and properties is given in section 2. Next, a comparison of the models with respect to computation time (computational effort) vs. error (accuracy) is provided in section 3. Finally, we conclude which model is best suitable for specific transformer geometry parameters based on our findings in the conclusion.

2 Overview of Leakage Inductance Models

The fundamental assumption in leakage inductance calculations is to assume equal but opposite magnetomotive forces $MMF$ of primary respectively secondary coil of the transformer. I.e. $N_1 \cdot i_1 = -N_2 \cdot i_2$ as depicted in Fig. 1.

![Fig. 1: Equivalent circuit of a lossless transformer with $MMF_1 = -MMF_2$. The magnetising current $i_m = 0$ and consequently, the leakage inductance $L_{\sigma}$ remains as serial inductance (here: related to the primary side)](image)

Further details on basic principles of leakage inductance calculations can be found in common literature (e.g. [12, 13]).

2.1 Common Assumptions of Models

All considered models are based on an inside-window cross section of the transformer as depicted in Fig. 2a, and Fig. 2b. Fig. 2a shows the considered transformer geometry, and defines the nomenclature in use. Fig. 2b defines the required geometry parameters. The current in the windings is perpendicular to the considered plane. This simplifies the transformer geometry to a straight and infinitely long arrangement of core and windings. Wires and foil conductors are unified to a rectangular winding block.

The leakage inductance per length $L_{\sigma}'$ (2D) is acquired with one of the models from the considered cross section in Fig. 2b. The leakage inductance $L_{\sigma}$ (3D) is finally obtained by scaling the per length inductance by the mean winding length $l_m$ according to (1).

$$L_{\sigma} = L_{\sigma}' \cdot l_m$$

The mean winding length of a 2-winding transformer can be calculated with (2) according to [14].

$$l_m = \pi \left( d_{\text{leg}} + 2f + a_1 + d + a_2 - \frac{a_2 - a_1}{2} \cdot \frac{a_1 + a_2 + 4d}{a_1 + a_2 + 3d} \right)$$

where $d_{\text{leg}}$ is the (equivalent) diameter of the central transformer leg. In case of rectangular center legs as depicted in Fig. 2d, $d_{\text{leg}}$ is calculated such that the cross section area is constant: $d_{\text{leg}} = 2 \sqrt{\frac{b_c \cdot d_c}{\pi}}$, where $b_c$ is the leg width and $d_c$ is the leg depth. Note that scaling the per length value by a mean length is effectively a second model that is introduced after acquiring a 2D-related value. This implies two significant simplifications:
Fig. 2: a)–b): Considered inside-window transformer geometry. a) Nomenclature definitions, b) Geometrical parameter definitions, c) Circular center leg, d) Rectangular center leg, e) Leakage field distribution; blue lines represent constant potential $|A_1(x,y)| = \text{const}$, respectively the magnetic flux $\vec{B}(x,y)$. The flux bend in horizontal direction inside the transformer window is termed flux-fringing effect, f) Transformer window with original windings and one layer of image windings in green. Factor $m^n$ is only used in Margueron model (see section 2.2 for details)

- The considered inside-window cross section is not valid along the whole winding length and
- The curvature of the field is neglected

This paper deals solely with leakage inductance per length models. Therefore, length related values acquired by 2D FEM simulations are taken as reference value for the accuracy of the models.

2.2 Categorisation of Models

All models are based on Ampère’s theorem as fundamental correlation between current and magnetic field (see Fig. 3). The assumed current densities are through-plane ($J_z(x,y)$), the resulting magnetic field is in-plane ($H(x,y)$, $B(x,y)$), whereas the magnetic vector potential $\vec{A}$ only has a through-plane component $A_z(x,y)$.

The models listed below have been considered because they are applicable to the considered transformer geometries in Tab. IV and relatively simple. These models can be categorised as follows:

- **1D models:**
  1D models assume purely axial leakage flux between windings of equal height. Thus, the leakage field depends on only the $x$-coordinate, but not on the $y$-coordinate.
  - Dowell [5]: Dowell derived the current density $J_z(x,\omega)$ by solving a second-order differential equation. Next, the voltage $V(\omega)$ across a portion of an arbitrary number of winding layers is derived. The impedance of the transformer is calculated by Ohm’s law. The leakage inductance is finally derived by $L_\sigma = \frac{\text{Im}(V(\omega)/I)}{\omega}$. In case of windings with different height, a winding height transformation is proposed in section 4.2.

- **2D models:**
  In 2D models, the leakage flux $\vec{B}(x,y)$ and the magnetic potential $A_z(x,y)$ are functions of both space coordinates $x$ and $y$. Consequently, the flux bending in horizontal direction at the bottom and the top of the windings is taken into account. This effect is termed “flux-fringing effect” and illustrated in Fig. 2e. All considered 2D models assume homogeneous current distribution in the windings. The transformer core is taken into account by replacing the core with an infinite number of current images (Rogowski, Roth), respectively a discrete number of current images (MGD, Margueron). Rogowski’s, Roth’s, and Margueron’s model are based on a solution of the Poisson
Fig. 3: Overview of analytical leakage inductance calculation models (yellow), fundamental assumptions (red), physical and mathematical derivation approaches (grey), and leakage inductance equations (green)

equation for the magnetic potential \( A_z \). A detailed and comprehensive explanation on this can be found in [15, chapter 5]. The considered 2D models are listed below:

- **Rogowski** [6]: Rogowski mirrored the original windings across the vertical transformer window edges and replaced the transformer legs with an infinite number of image windings in \( x \)-direction. The remaining spatial current distribution is expressed as a Fourier series dependent of the \( x \)-coordinate (single space harmonics). With the current distribution, a magnetic potential \( A_z(x,y) \) is derived from Poisson’s equation (see Fig. 3) valid within the winding-domain. For the domains without windings, Rogowski derived a potential \( A_z(x,y) \) from the Laplace equation with continuity boundary conditions at the domain interfaces. Finally, the magnetic energy is computed with \( W_m = \frac{1}{2} \iint_{A} A_z \cdot J_z \, dA \), and the leakage inductance is obtained with \( L_S = \frac{2W_m}{\mu_o} \).

In Rogowski’s leakage inductance equation, the Fourier series is replaced by a finite term. This term is subject to certain geometrical specifications: ●Windings of equal height, ●No gaps between windings and legs (i.e. \( e = f = 0 \) in Fig. 2b). In case of interleaved windings: ●Constant air gaps between windings and ●Windings of constant width.

In case of windings with different height, the proposed height transformation in section 4.2 needs to be executed before applying the actual model.

- **Roth** [16, 17]: Édouard Roth used a double Fourier series dependent of both, \( x \)- and \( y \)-coordinate (double space harmonics) to describe the current density distribution of each conductor [16]. Next, a potential \( A_z(k,x,y) \) for each conductor \( k \) is derived with Poisson’s equation. The total potential \( A_z(x,y) \) is calculated by superposing each potential \( A_z(k,x,y) \) of all conductors \( (A_z = \sum_{k=1}^{n} A_z(k,x,y)) \), where \( l \) is the number of conductors. Analogously to Rogowski, the leakage inductance is derived via the magnetic energy. A compact derivation of the leakage inductance from Roth’s potential vector was published in [17]. An insightful and compact summary of Roth’s work can be found in [18].

- **Margueron** [7, 19]: Margueron introduced a model that is based on a magnetic potential and the mirroring method. This model is the only considered model that takes the finite permeability of the magnetic core into account. Margueron assumed an infinitely thin (i.e. filamentary) and infinitely long current source at the center of each conductor \( k \). The potential \( A_z \) that results from this source is integrated over the infinitely long respective conductor.
This yields an analytical expression for the potential caused by conductor \( k \ A_z,k(x,y,a,b) \) (PEEC formula), where \( 2a \) is the conductor width and \( 2b \) is the conductor height. Note that this derivation has already been conducted in [20] for a single rectangular conductor. In Margueron’s model, all conductor potentials \( A_z,k \) are superposed to obtain the total potential \( A_z(x,y) \). The boundary conditions resulting from the magnetic core are replaced by a discrete number of image layers of the original conductors, that carry current of the magnitude \( I \cdot m^n = I \left( \frac{\mu_r-1}{\mu_r+1} \right)^n \), where \( I \) is the original current, \( n \) is the number of the respective image layer, and \( m \) is the factor taking the finite permeability of the core \( \mu_r \) into account (see Fig. 2d).

The leakage inductance is obtained by numerically integrating the energy density. In [19], Margueron improved his model by carrying out the integration analytically using a primitive function for the potential \( A_z,k(x,y,a,b) \).

Note that Lambert [8, 21] proposed the same model with a constant offset of the potential, a multi-winding geometry, and an inductance matrix referred to each winding couple. The two 2D-models used in [11] are also mathematically equivalent to Margueron’s model. A detailed derivation of the potential function can be found in [15, chapter 5.2.2].

- **Mean Geometric Distances (MGD)** [22, 23]: The MGD model simplifies the winding blocks to winding filaments. These winding filaments carry filamentary currents that cause circular magnetic fields. Self inductances \( L_s \) and mutual inductances \( M \) of the resulting filamentary coils are directly derived from Ampère’s law. Self inductances \( L_s \) are calculated using the mean geometric distance of a winding to itself, whereas mutual inductances \( M \) are calculated using geometrically averaged distances of the windings. The leakage inductance is obtained by \( L_\sigma = L_s - M \). This approach was firstly introduced by J. C. Maxwell in his famous treatise [22, chapter 691], and expanded to leakage inductance by Petrov [23, 24].

The categorisation above is illustrated in Fig. 3. Each model has properties that imply advantages and disadvantages. These qualitative properties are listed in Tab. I.

<table>
<thead>
<tr>
<th>Property</th>
<th>Dowell</th>
<th>Rogowski simple</th>
<th>Rogowski complete</th>
<th>Roth</th>
<th>Margueron 2007</th>
<th>Margueron 2010</th>
<th>MGD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H(x,y) ) acquirable</td>
<td>✓, ( H(x) )</td>
<td>-</td>
<td>✓</td>
<td>✓</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( H(x,y) ) necessarily acquired (computationally effortful)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>✓</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Equation for resistance ( R )</td>
<td>✓</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Finite ( \mu_{r,core} ) considered</td>
<td>-</td>
<td>(-), *</td>
<td>(-), *</td>
<td>-</td>
<td>✓</td>
<td>✓</td>
<td>-</td>
</tr>
<tr>
<td>Extendable to more than two windings</td>
<td>✓, **</td>
<td>✓, **</td>
<td>✓, **</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Frequency dependency considered</td>
<td>✓, ( J(x) )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Spatially distributed current fully considered</td>
<td>✓, (1D)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Accuracy improvable (with higher comp. effort)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>✓, **</td>
<td>✓, ***</td>
<td>✓, ****</td>
<td>✓, ****</td>
</tr>
</tbody>
</table>

*Originally: \( \mu_{r,core} \) of yokes considered. However, \( \mu_{r,core} \) of whole core required to increase accuracy according to 2D FEM results

**Constraints: • Constant distances between windings (interlayer gaps) • Fixed width of windings

***by: Increasing the number of sum index

****by: • Adding more layers of images and • Splitting coils into more than one filament
3 Comparison of Leakage Inductance Models

3.1 Error vs. Calculation Time of Models

Fig. 4 shows the model error (accuracy) vs. the calculation time (computational effort) of the previously introduced models that have been applied to three E-core transformers (No.1–3), and three U-core transformers (No.4–6). Note that small error represents high accuracy and short calculation time represents low computational effort. Hence, the ideal model marker would be located at the origin, representing 100% accuracy with almost no computational effort. To exclusively evaluate the accuracy of the 2D models, the leakage inductance per length of the inside-window scenario calculated by 2D FEM was taken as reference to determine the error.

- **Roth's model** achieves the best tradeoff between accuracy and computational effort in the general case. The model is the most accurate in every considered geometry. The error tends towards 0, as the analytical solution takes both, the distributed current density, and the magnetic core \((\mu_{\text{core}} \to \infty)\) into account. The computational effort of the model is also remarkably small considering the calculation times of around 0.2 ms.

- **Rogowski's model** is a viable solution for optimisations when it comes to geometries with rather small winding-leg distances. Transformers No.1–3 are exemplary for this, as the error is below 1%
in these scenarios. The small computational effort of Rogowski’s formula is the biggest advantage of the model, which is due to the compactness of the model’s closed form leakage inductance formula. Calculation times are as low as approximately 5 µs – 10 µs.

- **Margueron’s model** yields below 5% error in all considered geometries, which makes the model versatile in geometry. Note that the analytical integration of the energy integral according to the Margueron 2010 model [19] decreases the computational effort of the model significantly by about three orders of magnitude. The Margueron 2007 model [7] is much slower because a discrete number of potential values all over the conductors need to be calculated. Margueron’s model has an important advantage over the other models: The finite magnetic permeability of the core \(\mu_c\) is taken into account. This is essential, when it comes to cores with low permeability. Similar to the MGD model, the accuracy can be further increased by adding more layers of images.

- **The MGD model** is the second fastest model and yields errors below 3% in geometries with similar winding dimensions (i.e. \(a_1 \approx a_2\) resp. \(h_1 \approx h_2\)). Transformer No.1, and 4–6 are examples for this. The MGD model is not suitable for geometries with high relative height differences \(h_1 \approx h_2\), in which the error becomes significantly high (No.3: 9%, No.2: 6%). The accuracy of the model can be increased by adding more layers of images.

- **Dowell’s model** is by far the most inaccurate because it neglects the non-axial part of the leakage field between the windings. The error reaches 16% in the most critical case (transformer No.5). Also the computational effort is remarkably high. The model is slow because of the rather complex terms that take into account the frequency. However, as the magnetostatic inductance is already erroneous, the frequency dependency does not significantly improve the model.

The models were computed in Matlab R2018b on a Windows-10 notebook with 16 GB RAM and an Intel Core i7-8550U 4-core processor. The models were executed using only one core. The calculation routine was the following: Read out input data of irrelevant transformer \(\rightarrow\) Read out input data of relevant transformer \(\rightarrow\) Model \(A\) + time extraction \(\rightarrow\) Extensive optimisation script \(\rightarrow\) Model \(B\) + time extraction \(\rightarrow\) Extensive optimisation script \(\rightarrow\) … … This represents a realistic calculation routine when the model is implemented within an optimisation. Another reason for this implementation is that the calculation time constantly decreased from computation to computation when only running the leakage inductance models. After several thousand calculations, the computation time of the respective model was by several orders of magnitude lower compared to the first time. This was due to system and script specific optimisations by Matlab/Windows that cannot be controlled by the user. In a converter optimisation, such performance optimisations are not realistic since the parameters change with every iteration step.

The calculation routine was executed 100 times and the first 10 measured time values were excluded because the extracted times were highly volatile. The mean time of the remaining 90 times was calculated and used for the comparison. All models were implemented in series so that the pure computational effort of the models is compared. Note that the absolute time values depend on the implementation and the system. Therefore, relative time comparisons are more significant than the absolute values.

As most models assume infinite permeability of the core, the core permeability \(\mu_{\text{core}}\) of all transformers No.1–6 was assumed to be infinite in the conducted calculations. This assumption is suitable for comparing the models’ validity and accuracy. Furthermore, 2D FEM simulations have shown that the difference in resulting leakage inductance is negligibly small for permeabilities greater than \(10^3\). Therefore, the models, and FEM simulations were computed with \(\mu_{\text{core}} = 10^9\).

In some models, fixed integers need to be set. An increase of this integer leads to higher accuracy but also higher computational effort. Roth’s model was computed with a sum index of the Fourier series of \(j = 25\). Both, Margueron’s model and the MGD model were executed with one image layer. Margueron 2007 was computed with a resolution of \(200 \times 200\) \(x\)- and \(y\)-coordinates. These values were set such that a reasonable tradeoff between computation time and error was achieved.

The resulting 3D inductances of the models are compared to measurements and 3D FEM results in Tab. II. This table shows that the use of the mean winding length introduces additional error into the cal-
The different per length inductances of inside-window and outside-window cross sections as demonstrated in [11, 25] need to be considered to achieve better overall accuracy.

Table II: Measured leakage inductance values of transformers No.1–6 and resulting errors of 3D FEM, and leakage inductance models

<table>
<thead>
<tr>
<th>Measurement $L_{\sigma}$</th>
<th>3D FEM</th>
<th>Dowell</th>
<th>Rogowski simple</th>
<th>Rogowski complete</th>
<th>Roth</th>
<th>Margueron</th>
<th>MGD</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. 1 26.90 µH</td>
<td>2.7%</td>
<td>5.6%</td>
<td>-5.2%</td>
<td>-2.2%</td>
<td>-2.3%</td>
<td>2.2%</td>
<td>-0.8%</td>
</tr>
<tr>
<td>No. 2 10.80 µH</td>
<td>-3.6%</td>
<td>-0.9%</td>
<td>-9.3%</td>
<td>-8.2%</td>
<td>-7.4%</td>
<td>-4.5%</td>
<td>-13.2%</td>
</tr>
<tr>
<td>No. 3 13.50 µH</td>
<td>2.5%</td>
<td>8.3%</td>
<td>-3.9%</td>
<td>-2.0%</td>
<td>-1.6%</td>
<td>2.3%</td>
<td>-10.6%</td>
</tr>
<tr>
<td>No. 4 10.90 µH</td>
<td>-3.7%</td>
<td>7.4%</td>
<td>-0.3%</td>
<td>-0.2%</td>
<td>-3.9%</td>
<td>-1.6%</td>
<td>-3.8%</td>
</tr>
<tr>
<td>No. 5 5.35 µH</td>
<td>4.6%</td>
<td>29.5%</td>
<td>13.2%</td>
<td>14.3%</td>
<td>11.4%</td>
<td>15.6%</td>
<td>14.1%</td>
</tr>
<tr>
<td>No. 6 26.60 µH</td>
<td>5.6%</td>
<td>28.4%</td>
<td>13.0%</td>
<td>14.3%</td>
<td>11.3%</td>
<td>15.6%</td>
<td>12.4%</td>
</tr>
</tbody>
</table>

3.2 Geometrical Limitations of Models

Each model relies on simplifications that inevitably introduce error into the calculation. The extent of this error depends on the model and the transformer geometry. The most significant errors in Fig. 4 can be explained by the model simplifications and the geometry. Therefore, dimensionless geometrical characteristic numbers have been defined in Tab. III to link model errors of the leakage inductance per length with specific geometrical properties of the transformer geometries.

Table III: Left-hand side: Geometrical characteristic numbers of transformers (see Fig. 2b for parameter definitions), right-hand side: Model errors of inside-window leakage inductance per length $L_{\sigma}$

<table>
<thead>
<tr>
<th>Char. number</th>
<th>Rel. height difference $\Delta h_{rel}$</th>
<th>Horizontal gap ratio $\delta_{h_{ob}}$</th>
<th>Porosity factor $\eta$</th>
<th>Model error (reference: $L_{\sigma}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equation</td>
<td>$h_{max} - h_{min}$ $h_{max}$</td>
<td>$e = f$ $w_{w}$</td>
<td>$b + g$ $w_{w}$</td>
<td>Dowell</td>
</tr>
<tr>
<td>No. 1</td>
<td>0.11</td>
<td>0.20</td>
<td>0.06</td>
<td>8.04% -3.01% 0.05% -0.01% 4.61% 1.46%</td>
</tr>
<tr>
<td>No. 2</td>
<td>0.15</td>
<td>0.36</td>
<td>0.07</td>
<td>6.99% -2.13% -0.84% -0.01% 3.15% -6.30%</td>
</tr>
<tr>
<td>No. 3</td>
<td>0.22</td>
<td>0.36</td>
<td>0.06</td>
<td>9.99% -2.41% -0.41% -0.01% 3.92% -9.18%</td>
</tr>
<tr>
<td>No. 4</td>
<td>0.00</td>
<td>0.48</td>
<td>0.29</td>
<td>11.73% 3.78% 3.80% -0.04% 2.39% 0.14%</td>
</tr>
<tr>
<td>No. 5</td>
<td>0.00</td>
<td>0.40</td>
<td>0.25</td>
<td>16.13% 1.51% 2.50% -0.05% 3.69% 2.32%</td>
</tr>
<tr>
<td>No. 6</td>
<td>0.08</td>
<td>0.28</td>
<td>0.22</td>
<td>15.31% 1.48% 2.65% -0.01% 3.85% 0.99%</td>
</tr>
</tbody>
</table>

*related to primary winding

- Roth’s and Margueron’s models are the most versatile, as the model error does not significantly depend on the transformer geometry.

- Rogowski’s model results in high errors for both, horizontal gap ratios $\delta_{h_{ob}}$, and porosity factors $\eta$ (see Tr. No.4–6). Nonzero values of $b$ and $g$ are taken into account, however with the assumption $e = f = 0$. Therefore, considering $b$ and $g$ merely increases the accuracy of the model. It is noteworthy that the proposed height transformation in section 4.2 does not introduce considerable error in the inspected cases (see Tr. No.1–3). However, it leads to better agreement of the Rogowski simple model compared to the Rogowski complete model in tr. No.5–6.

- The MGD model is not suitable for windings with different dimensions. This can be observed in Tr. No.2 ($\approx 6$% error) and No.3 ($\approx 9$% error) which feature considerably high relative height differences $\Delta h_{rel}$.

- Dowell’s model yields significant errors for high porosity factors $\eta$ i.e. when winding-yoke distances $b$ and $g$ are high with respect to the window height $h_{w}$. Dowell’s model assumes purely...
axial leakage field between the windings and neglects the flux bend towards the top and the bottom of the windings. However, this field character is only present when the windings reach the transformer yokes. Therefore, high values of $\eta$ increase the model error. Transformers No.4–6 are representative for this (error between 12% and 16%).

4 Conclusion

Several analytical 1D and 2D leakage inductance models have been compared and assessed with respect to a trade-off between accuracy and computational effort. Roth’s model [16] delivers the best accuracy combined with rather low computational effort. The error referred to the 2D FEM simulation was negligibly small, and the calculation times were as low as 0.2 ms on a standard notebook. The model is geometrically versatile as it is applicable to all geometries with rectangular windings within a rectangular transformer window, given that winding and window edges are parallel. As long as the distances between windings and core are small compared to the transformer window dimensions, Rogowski’s model [6] yields satisfactory accuracy (typically less than 1% in the simulated scenarios). Rogowski’s closed form leakage inductance formula is rapidly executable with calculation times of around $5\mu s - 10\mu s$. The model of Margueron [19] is the only considered model that takes the finite permeability of the core $\mu_{r,\text{core}}$ into account. Transformers with a low-permeability core are therefore best examined with this model. Furthermore, the error is constantly below 5%, regardless of the considered geometry. The accuracy of the model can be further improved by increasing the number of image layers (by the cost of additional computational effort). These properties make Margueron’s model the most versatile one.

Appendix

4.1 Transformer Parameters

Table IV: Parameters of considered transformers

<table>
<thead>
<tr>
<th>No.</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$d$</th>
<th>$h_1$</th>
<th>$h_2$</th>
<th>$h_3$</th>
<th>$b$</th>
<th>$g$</th>
<th>$e$</th>
<th>$f$</th>
<th>$b_c$</th>
<th>$g_c$</th>
<th>$d_c$</th>
<th>$d_{leg}$</th>
<th>$l_m$</th>
<th>Further data</th>
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<td>3.0</td>
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<td>52.0</td>
<td>44.0</td>
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<td>1.9</td>
<td>7.2</td>
<td>0.0</td>
<td>20.2</td>
<td>20.2</td>
<td>20.2</td>
<td>22.8* 111.1</td>
<td>E   rect. 23 26</td>
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</tr>
<tr>
<td>3</td>
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<td>4.5</td>
<td>7.0</td>
<td>48.9</td>
<td>37.9</td>
<td>5.5</td>
<td>1.5</td>
<td>1.5</td>
<td>8.5</td>
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<td>26.0</td>
<td>28.1</td>
<td>30.5* 142.3</td>
<td>E   rect. 19 18</td>
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<td></td>
</tr>
<tr>
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<td>4.3</td>
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<td>10.9</td>
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<td>12.2</td>
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</tbody>
</table>

*...equivalent diameter of center leg; **...yoke depth

4.2 Winding Height Transformation (Required for Dowell and Rogowski)

For transforming windings of different height into windings of equal height, it is proposed to proceed as following: The geometric mean height is calculated according to (3). The geometric mean was found to yield more accurate results than the arithmetic mean. The distances from windings to yokes $b$ and $g$ are corrected according to (4), such that the window height $h_w$ is the same as before.

$$h_{\text{geom}} = h = \sqrt{h_1 \cdot h_2}$$ (3)

$$b_{\text{corr}} = b + \frac{h_1 - h}{2} ; g_{\text{corr}} = g + \frac{h_1 - h}{2}$$ (4)

References


